Reachability and Boundedness in Timed HMSCs

K Narayan Kumar

Chennai Mathematical Institute
http://www.cmi.ac.in/~kumar

Joint work with P Gastin, Madhavan Mukund

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Overview

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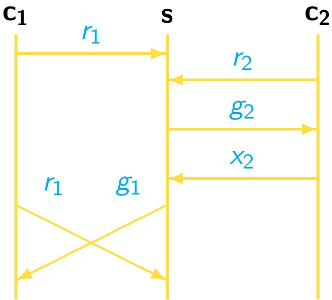
Can we extend the analysis techniques to the timed setting?

MSCs

Two clients and a server



MSCs as Partial Orders

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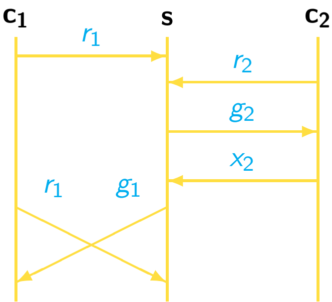

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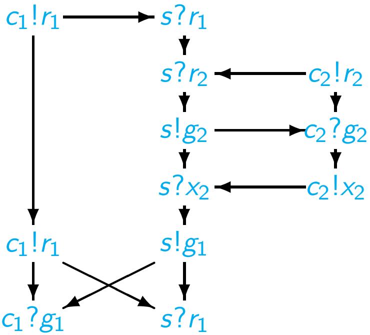

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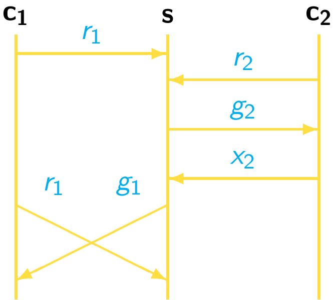

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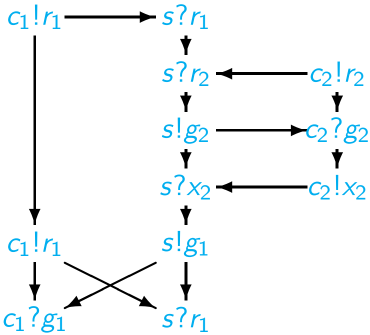
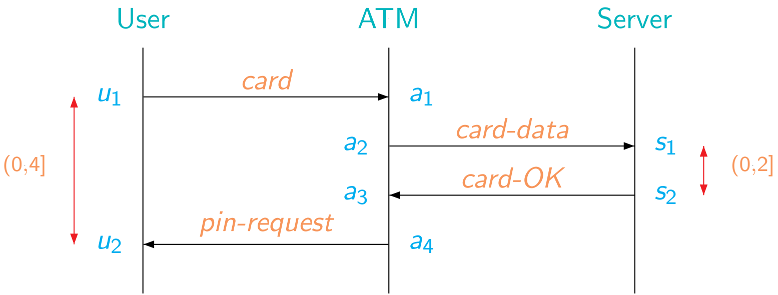


- All channels are assumed to be FIFO.
- An MSC can be regenerated from any one sequentialization.

MSCs with time constraints



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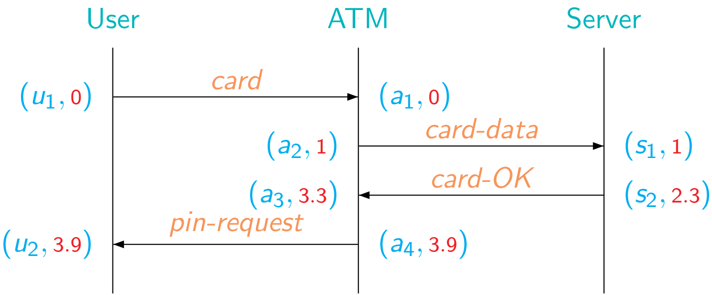
    - e is $p!q(m)$ and e' is corresponding receive $q?p(m)$

A timed behaviour



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- TC-MSC $T \Rightarrow L(T)$, set of timed MSCs that cover T

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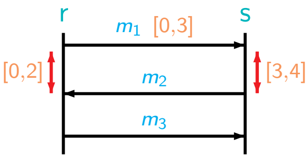

TC-MSCs and Timed MSCs

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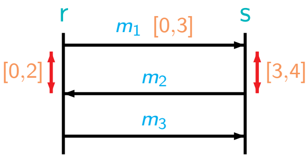
- A TC-MSC is said to be realizable if it is covered by atleast one timed MSC.



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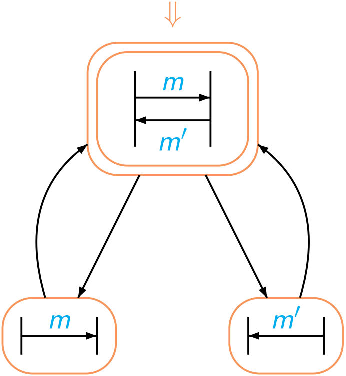

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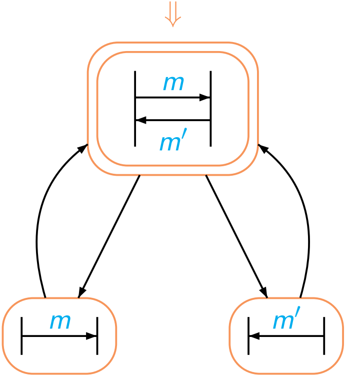

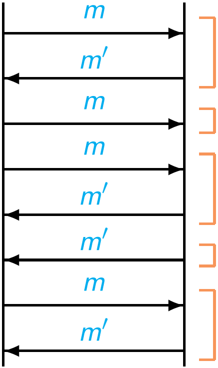

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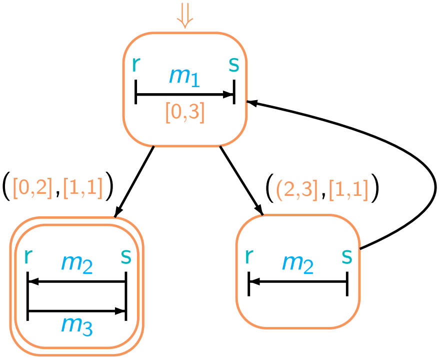

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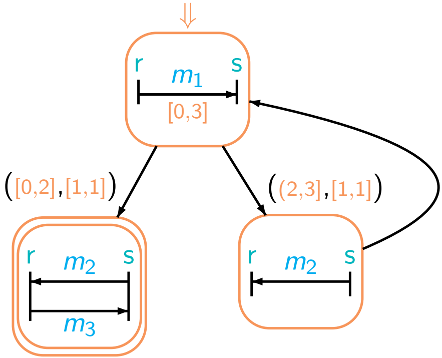

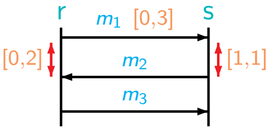

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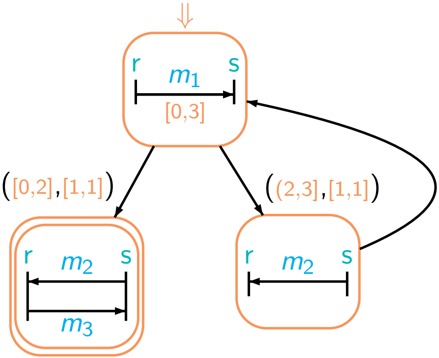

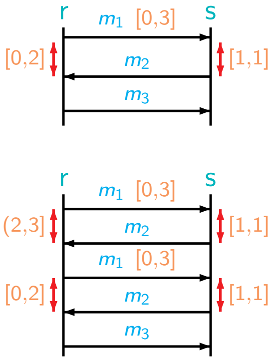

Reachability

A natural problem to consider is the following:

Given a TC-MSG G and a state q in G, does there exist a path $q_0 q_1 \ldots q_k = q$ from an initial state q_0 such that the TC-MSG generated by this path is realizable.

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This problem is trivial for ordinary MSGs.

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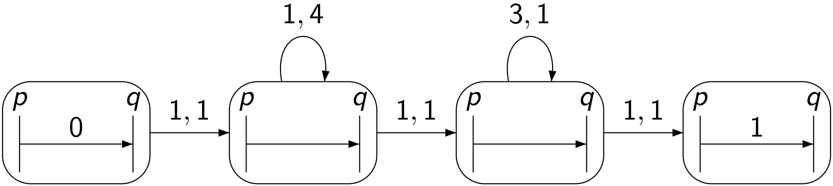

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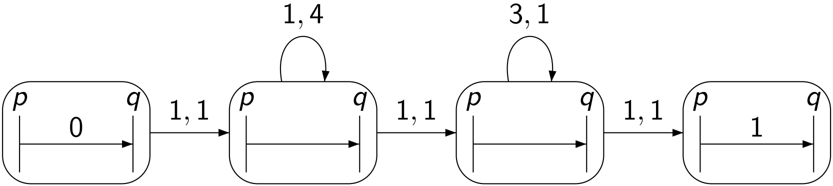

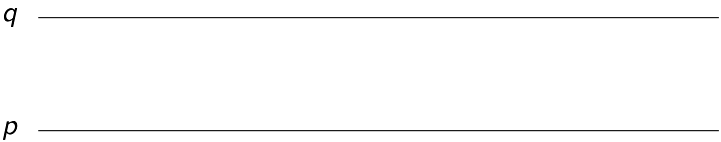

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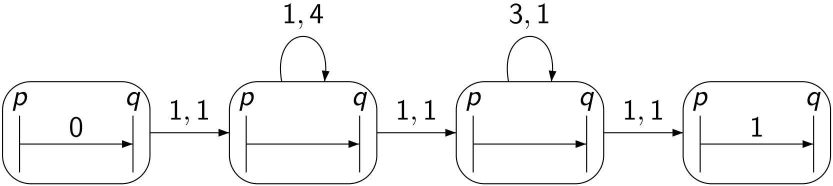

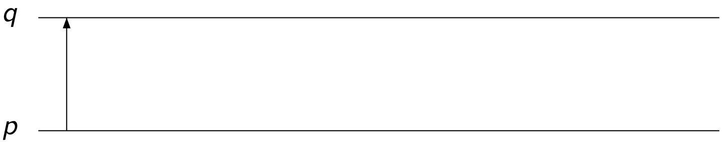

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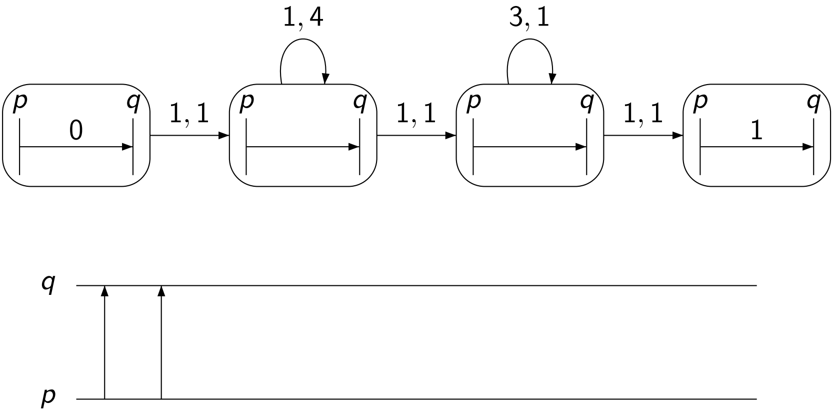

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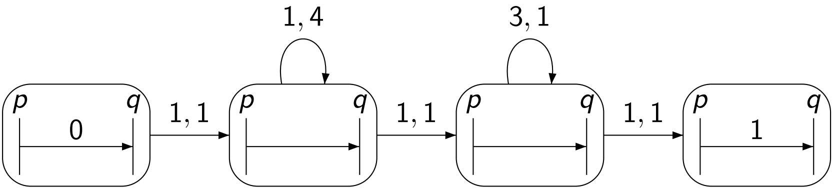

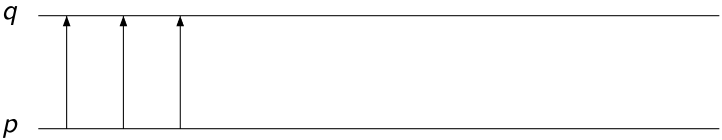

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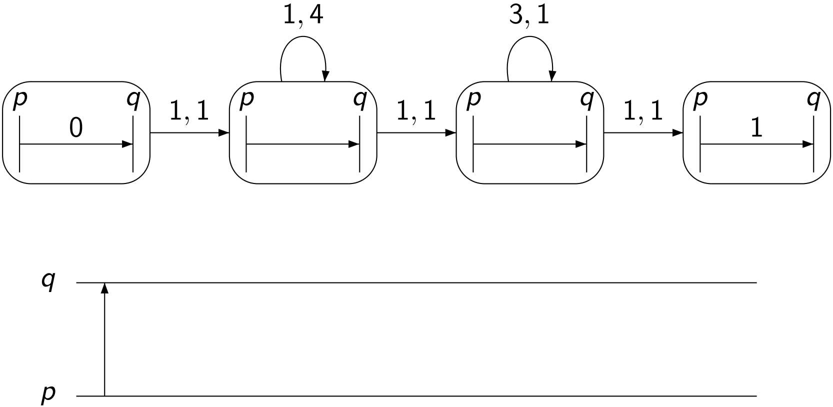

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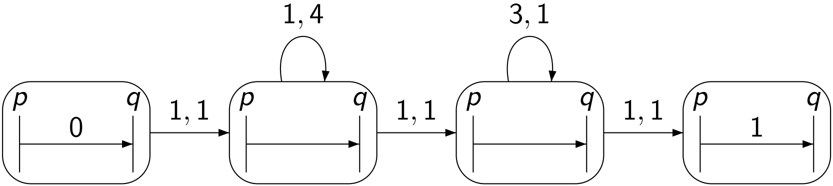

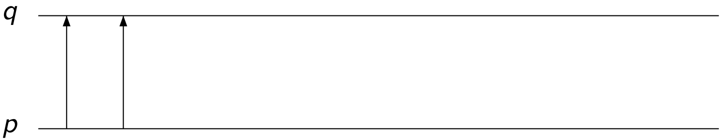

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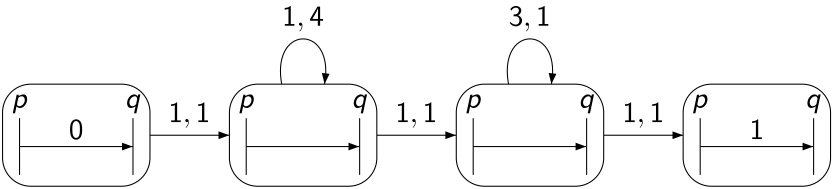

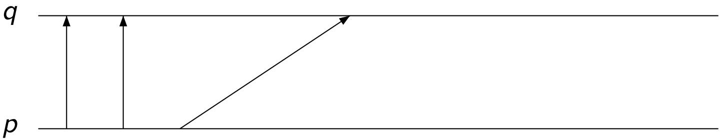

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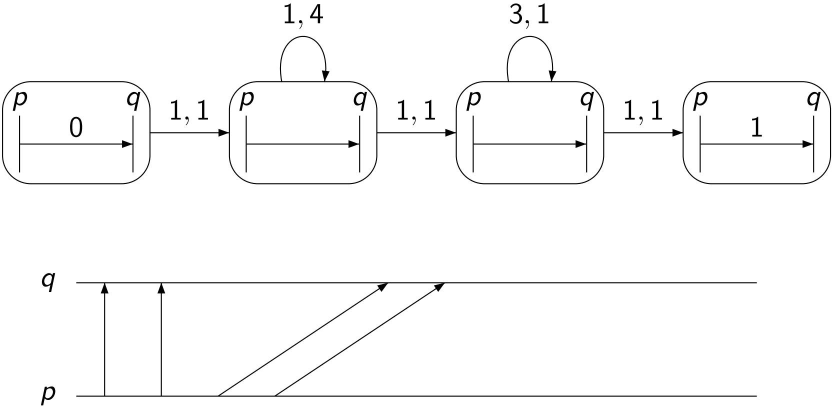

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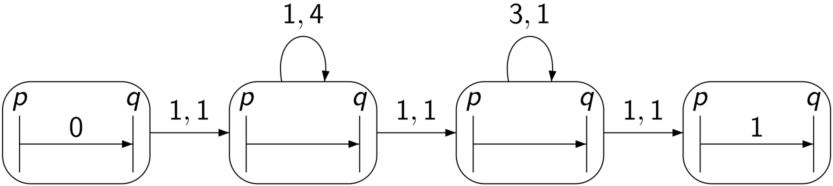

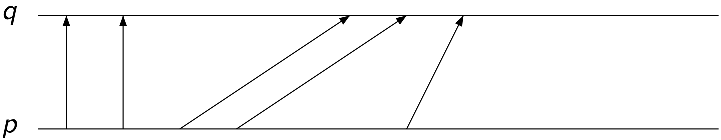

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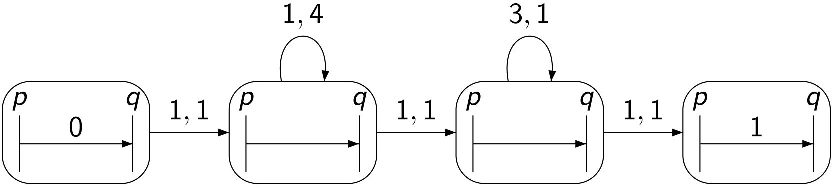

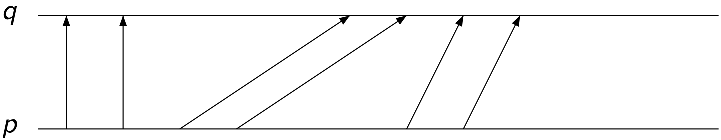

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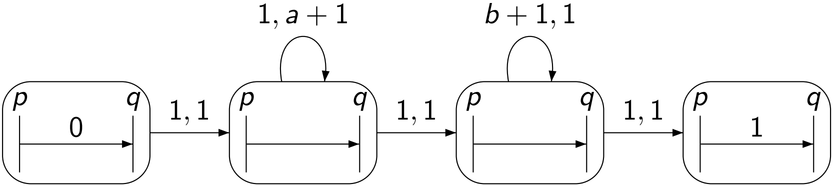

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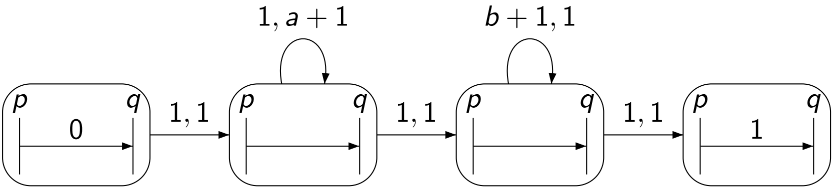

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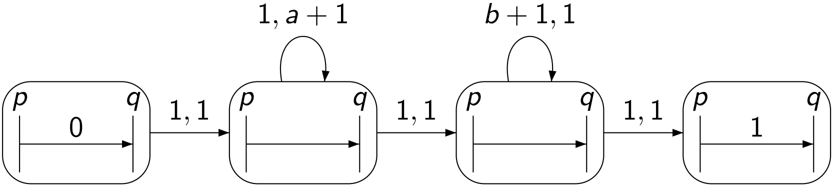

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- Simple paths may not be realizable while those with loops may be.

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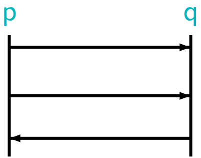

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- Universal boundedness is a decidable property of MSGs.

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A TC-MSC is (universally/existentially) B bounded if all its timed realizations are (universally/existentially) B bounded.

A TC-MSG is (universally/existentially) bounded if there is a B such that all the TC-MSCs realizing it are (universally/existentially) B bounded.

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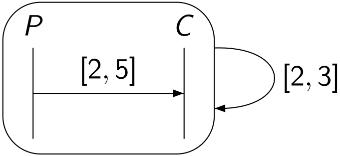

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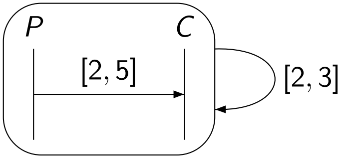

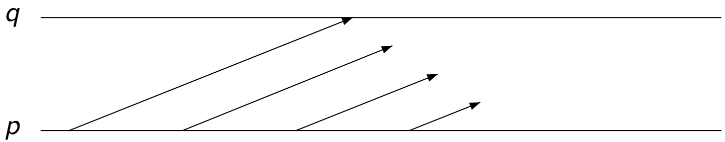

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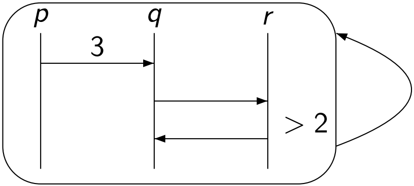

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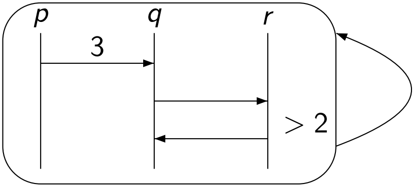

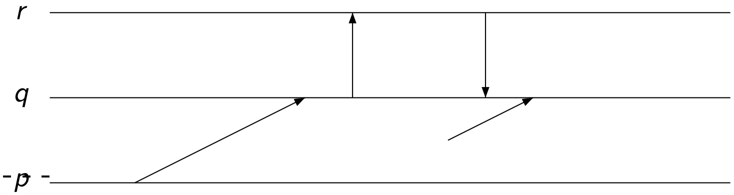

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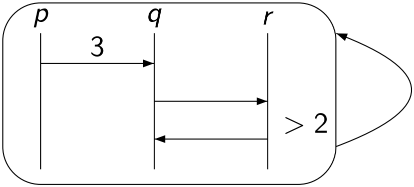

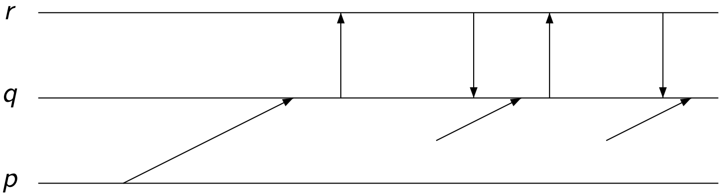

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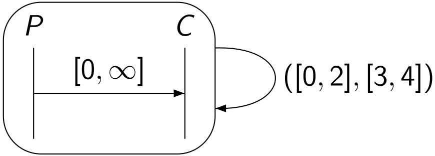

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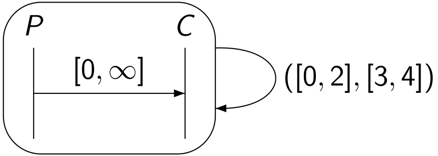

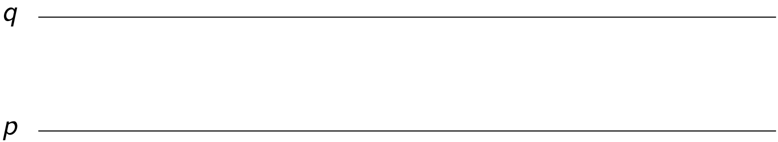

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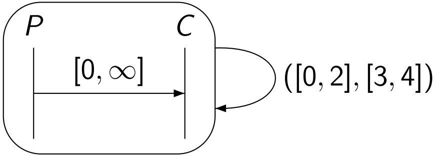

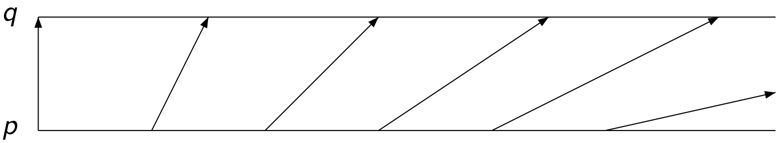

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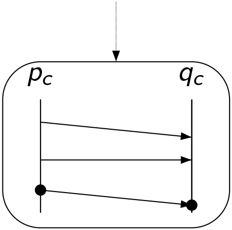

The reduction

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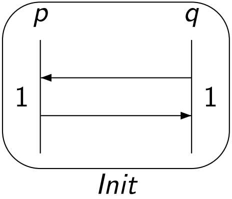

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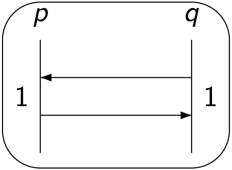

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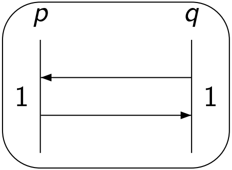

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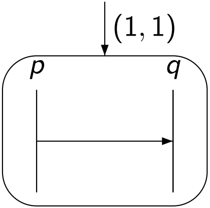

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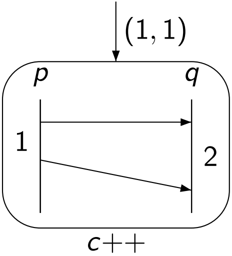

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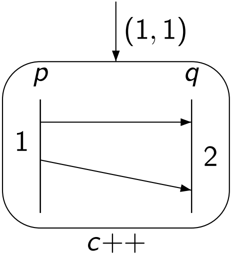

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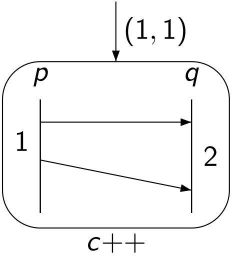

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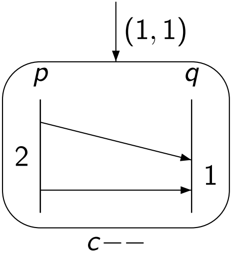

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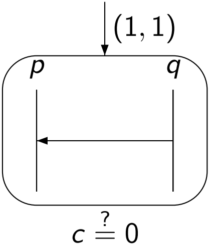

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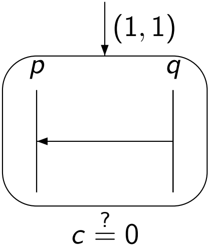

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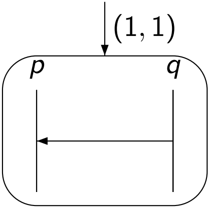

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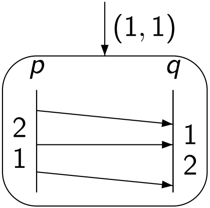

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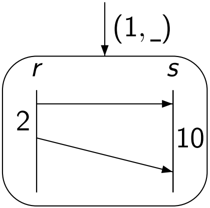

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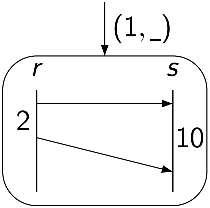

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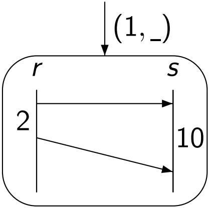

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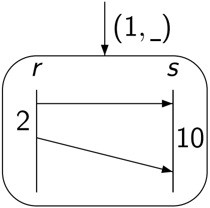

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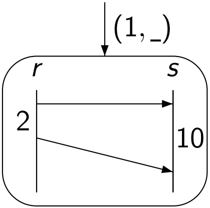

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Checking boundedness for TC-MSGs is undecidable

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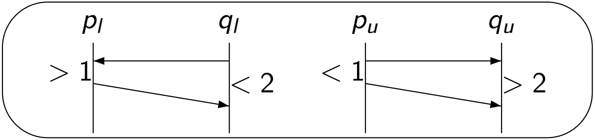

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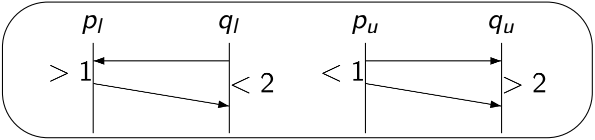
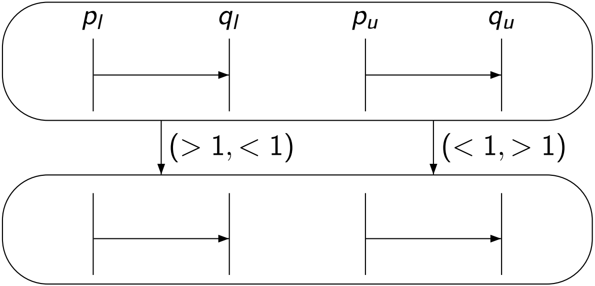


Composition between Nodes



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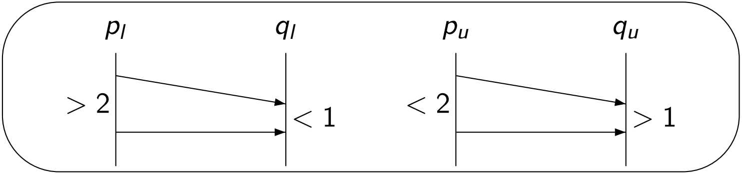

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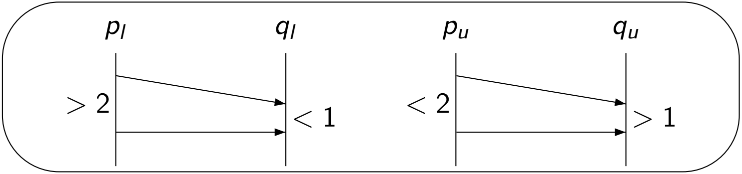

And so on ...

Stronger Undecidability – 2

What about the reachability problem for channel bounded TC-MSGs?

Stronger Undecidability – 2

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The reachability problem for channel bounded TC-MSGs is also undecidable.

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Even with the restriction that constraints across nodes are permitted only on a a fixed process, the reachability and boundedness problems for TC-MSGs remain undecidable.

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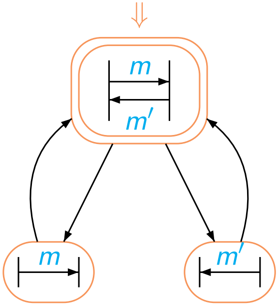

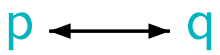

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