On the Membership (Word) Problem of Rational Trace Languages

Case of Local Well-Structured Loop-Only Automata

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Introduction

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- obtain a lower polynomial time complexity
- possibly not a function of the dependences
- at least for some rational trace languages of applicative interest (e.g. compilation)

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- this is a work in progress ...

Loop-Only Automaton

Automaton Model

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  union and Kleene star are forbidden

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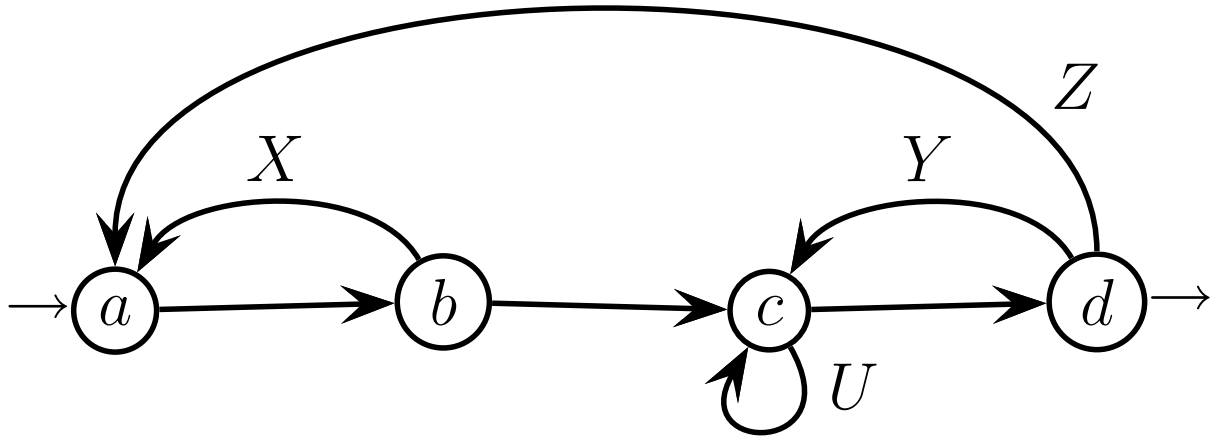

loop-only automaton over alphabet $\{a, b, c, d\}$

(by convention loop names X, Y, Z and U label the backward arcs)

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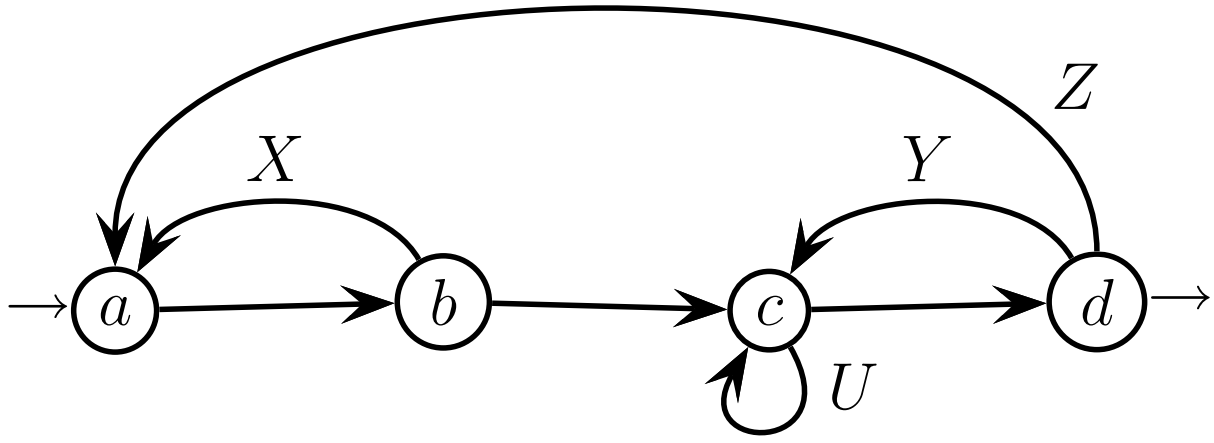

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$$\Big((\, a\, b\,)^{+}\ (\, c^{+}\, d\,)^{+} \Big)^{+}$$

linear regexp with concatenation and Kleene cross

Counterexamples

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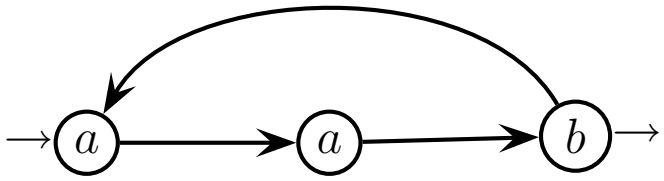

not local (counting)

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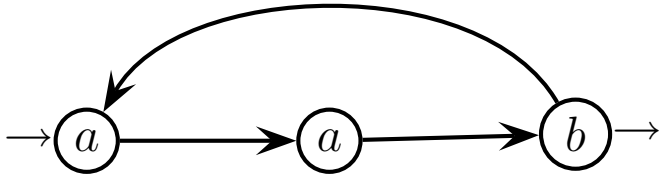

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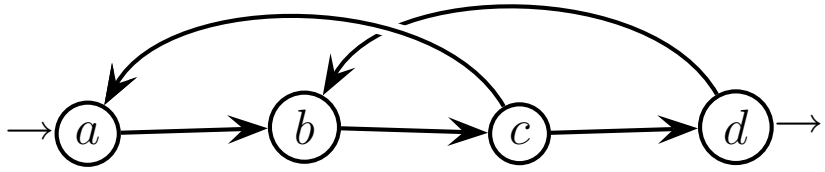

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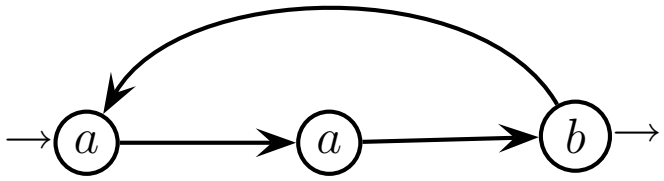

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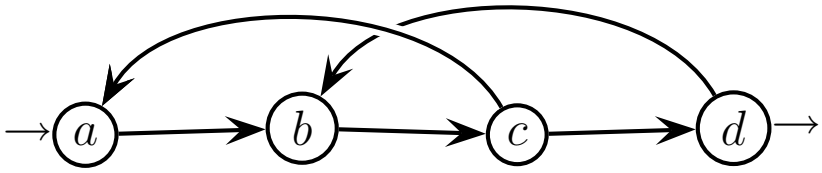

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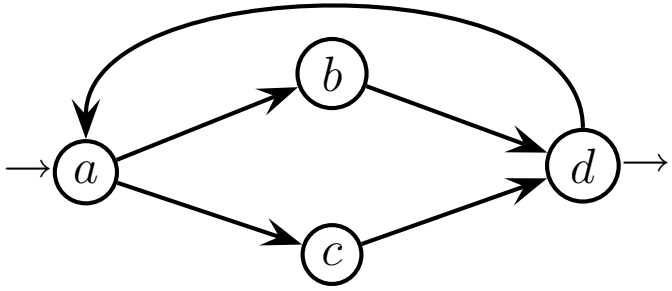

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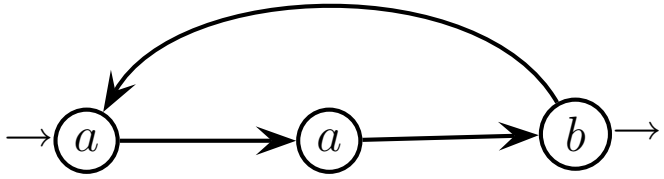

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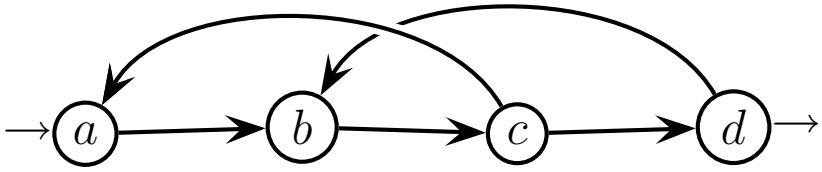

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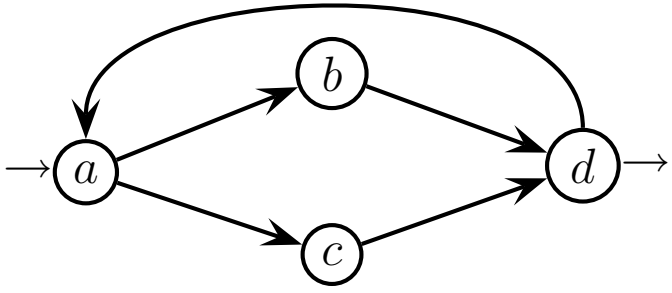

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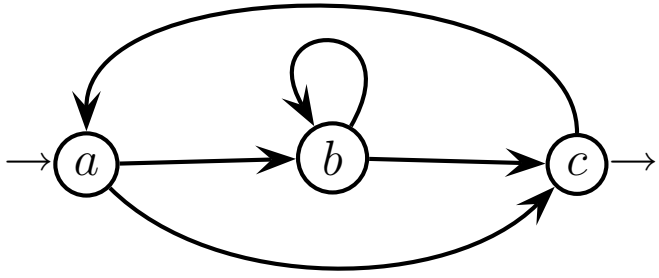

Kleene star

$$(\; a \; b^{*} \; c \;)^{+}$$

Hierarchy & Loop Identification

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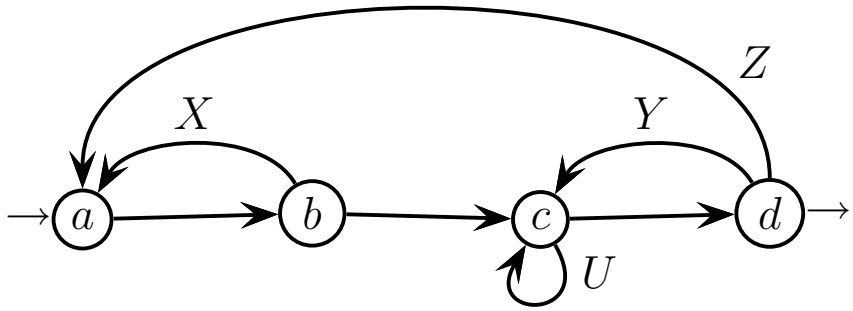

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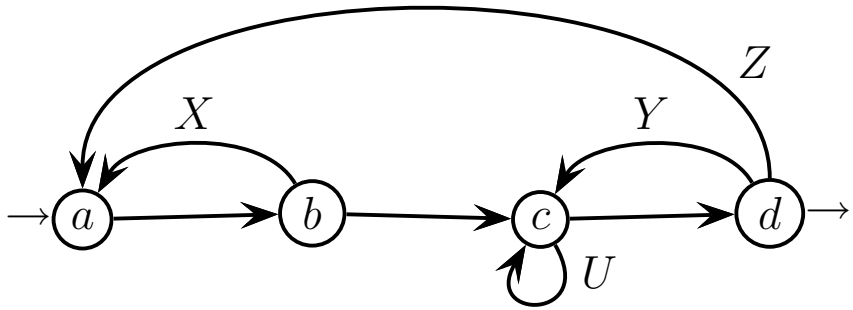

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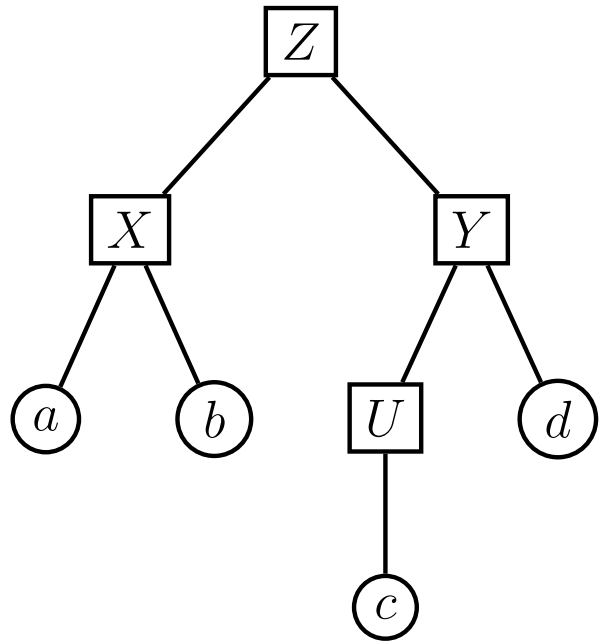

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hierarchy tree

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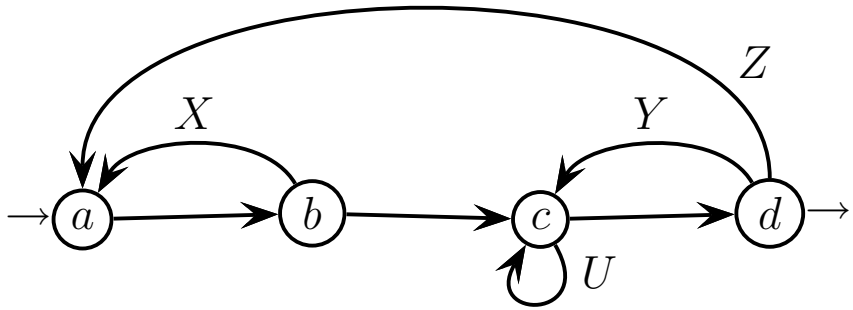

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loop	identification			
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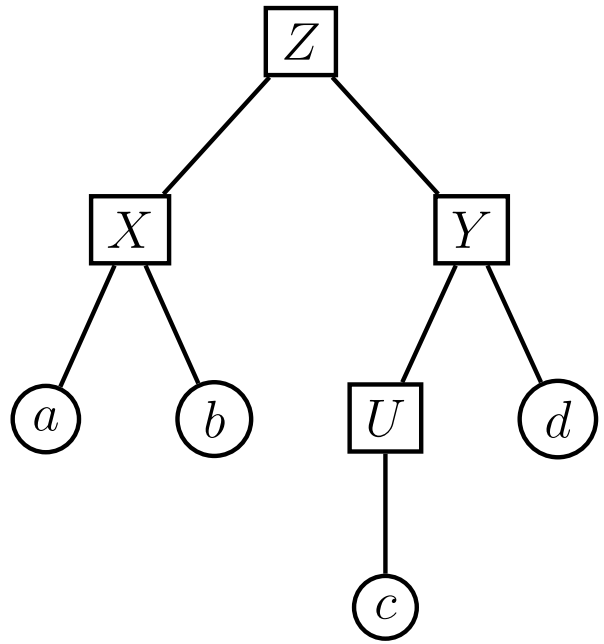
U	c			



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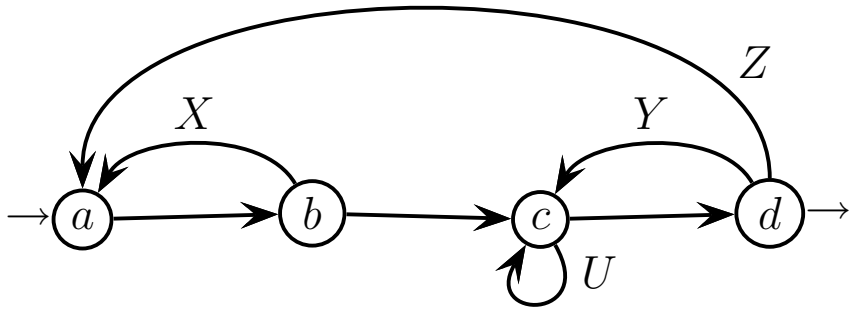

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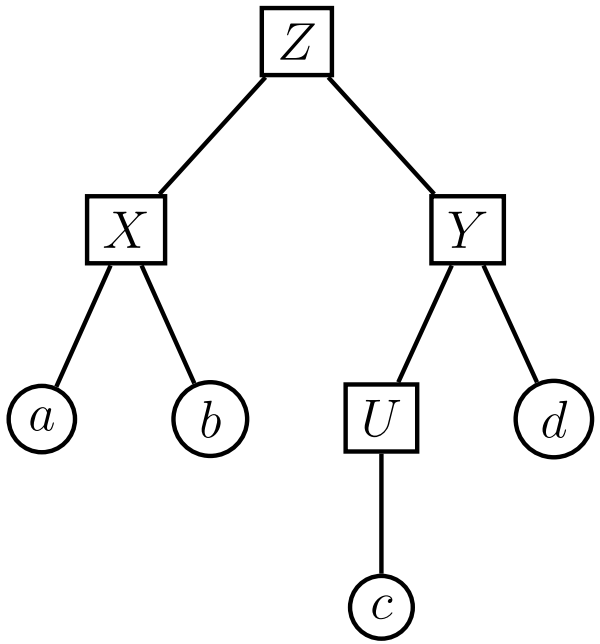

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every loop is identified (in general not uniquely)
by a single letter or at most by a letter pair

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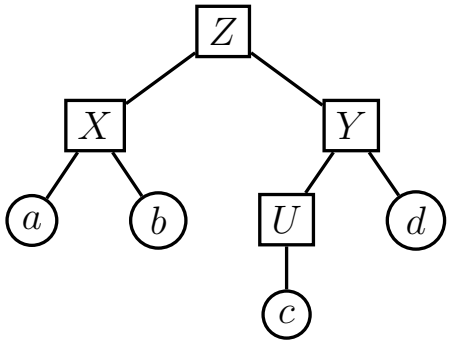

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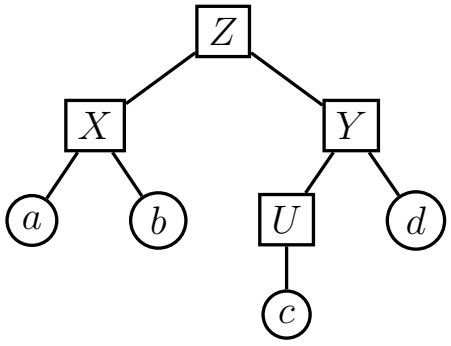

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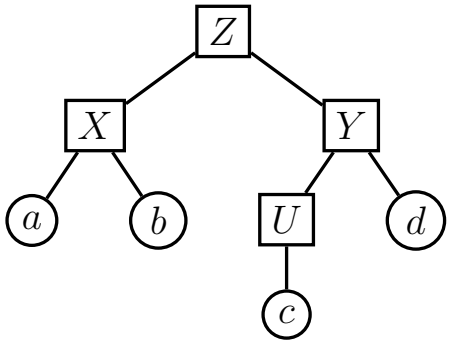

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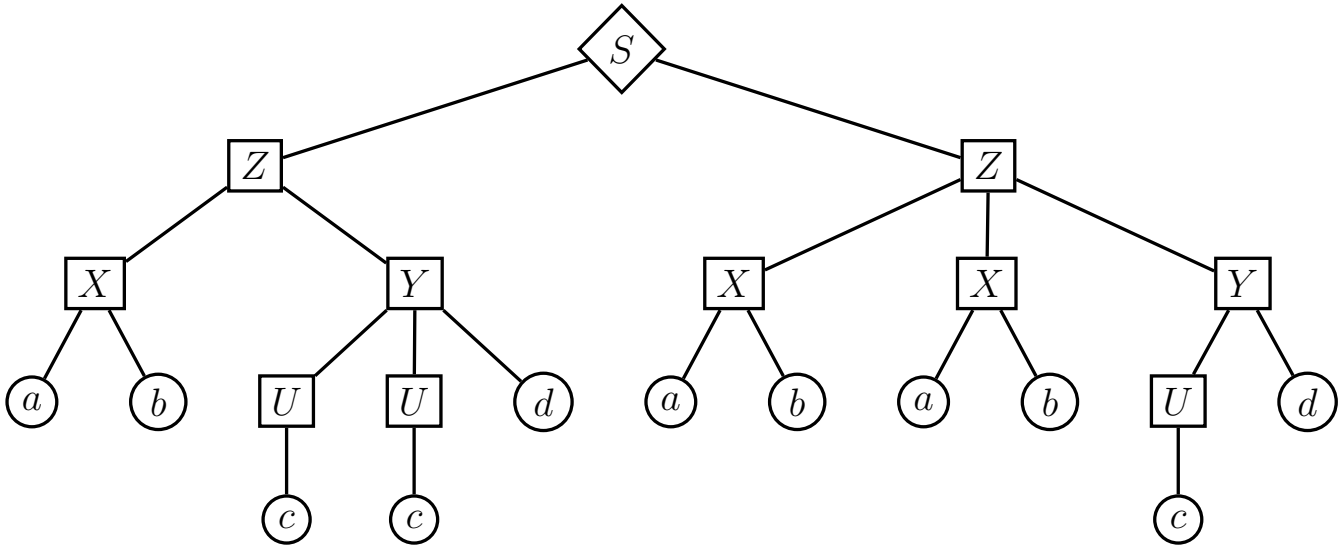

syntax tree of string x (S is a formal root)

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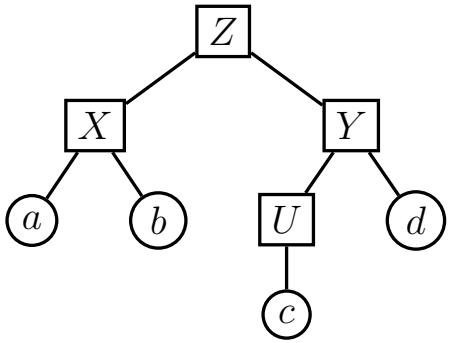

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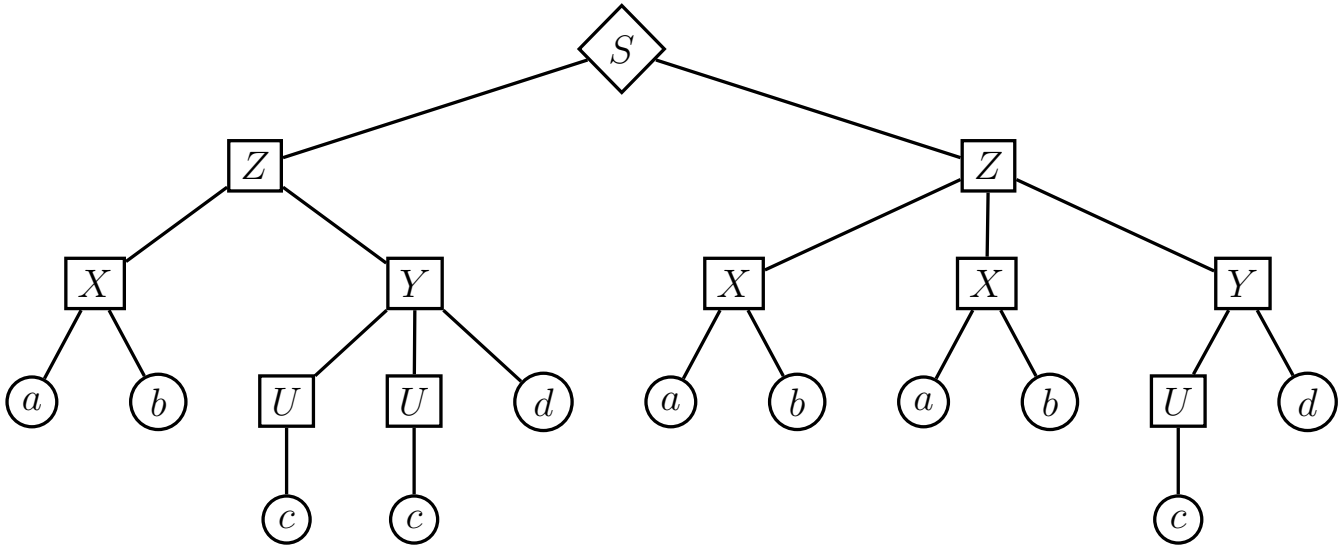

syntax tree of string x (S is a formal root)

string x has a syntax tree iff it is recognised by the automaton

Membership Problem

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- analyses in parallel input string x
- builds the syntax tree of string $y \cong_D x$
- demonstrated to work on several examples
- but misses a fully rigorous formal setting

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  - with a more rigorous complexity analysis
- potentiality for generalisation (union and star)

Simple Case

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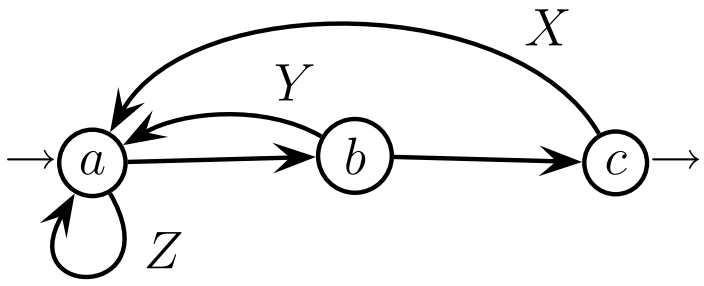

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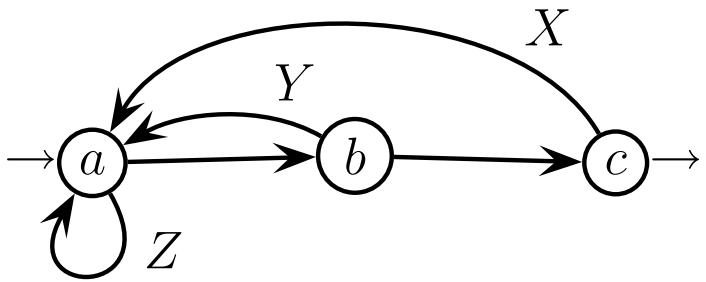

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$$\pi_{a\,b}(x) = a_1 \quad b_1 \quad a_2 \quad a_3 \quad b_2 \quad a_4 \quad a_5 \quad b_3$$

$$\pi_{a\,c}(x) = a_1 \quad a_2 \quad a_3 \quad c_1 \quad a_4 \quad a_5 \quad c_2$$

$$y =$$

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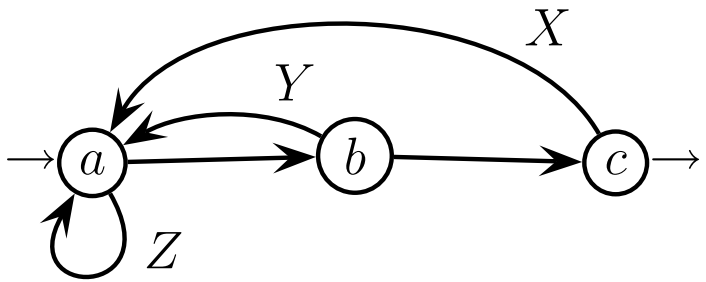

$x = a\,b\,a\,a\,c\,b\,a\,a\,c\,b$

dependent letters: $a\,b$ and $a\,c$

$$
\begin{array}{rcccccccc}
\pi_{a\,b}(x) & = & a_1 & b_1 & a_2 & a_3 & b_2 & a_4 & a_5 & b_3 \\
 & & - & & & & & & & \\
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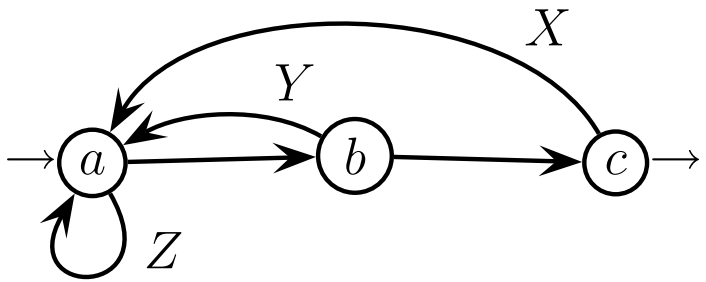

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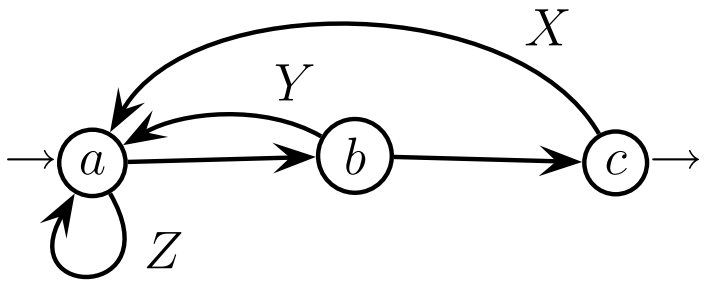

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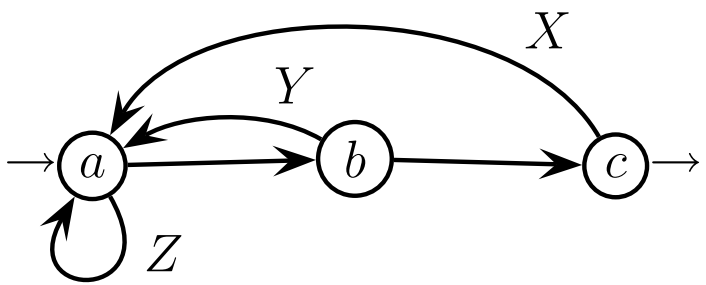

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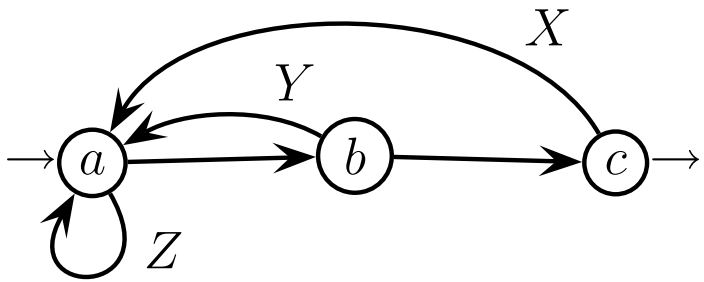

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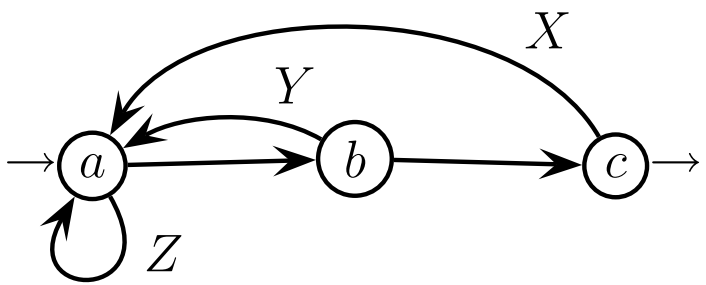

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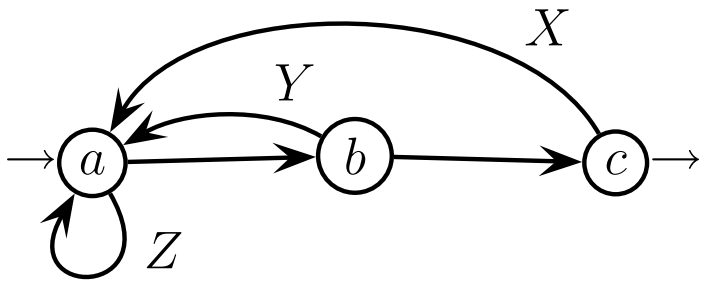

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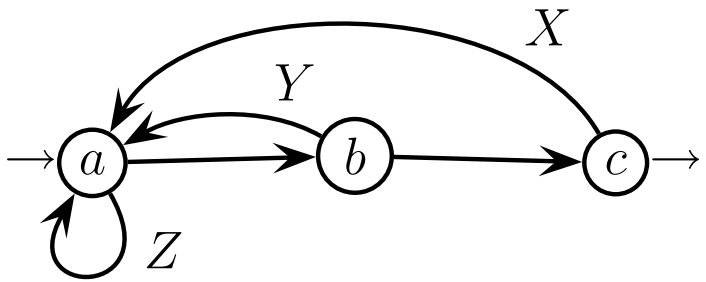

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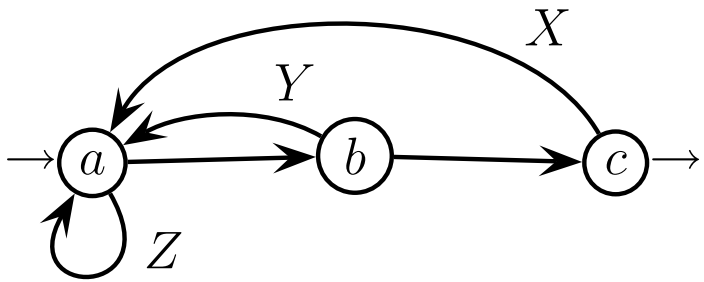

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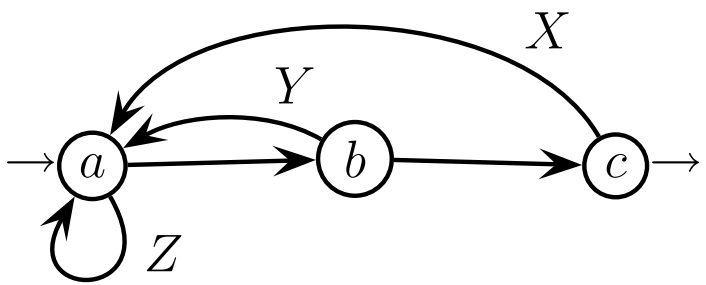

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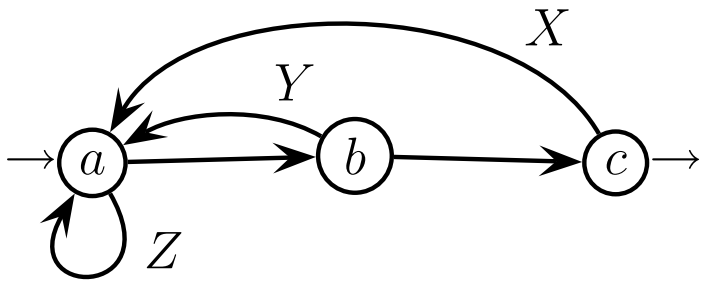

$x = a\,b\,a\,a\,c\,b\,a\,a\,c\,b$

dependent letters: $a\,b$ and $a\,c$

string $y \cong_D x$ with $y \in L\,(A)$ exists

string x is accepted

$$
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 & & - & - & - & - & - & - & - & - & \\
\pi_{a\,c}\,(x) & = & a_1 & a_2 & a_3 & c_1 & a_4 & a_5 & c_2 & & \\
 & & - & - & - & - & - & - & - & & \\
y & = & a_1 & b_1 & a_2 & a_3 & b_2 & c_1 & a_4 & a_5 & b_3 & c_2
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- is based on parallel scan of projections
- similar idea is used in the general case
  - examine in parallel projections
  - build bottom-up the syntax tree

  parallelism is exploited more extensively

Composition and Syntax Tree

Integer Composition

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- give automaton A and hierarchy tree H

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- χ_Y^X encodes the relationship between nodes X and Y

Example

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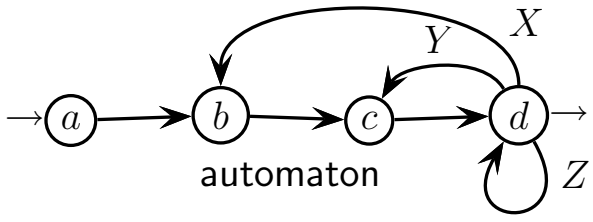

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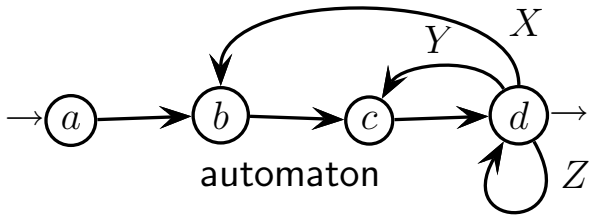
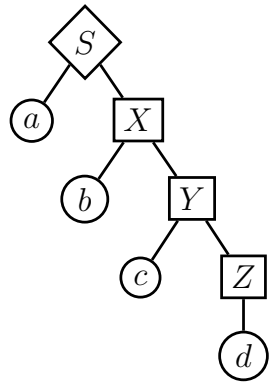
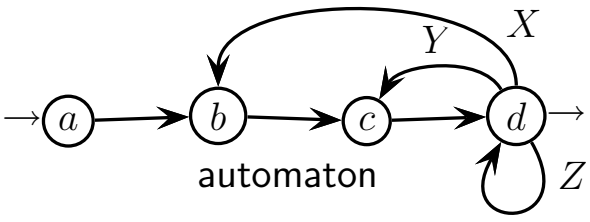


automaton



hierarchy tree

Example



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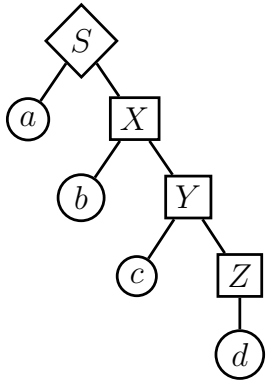

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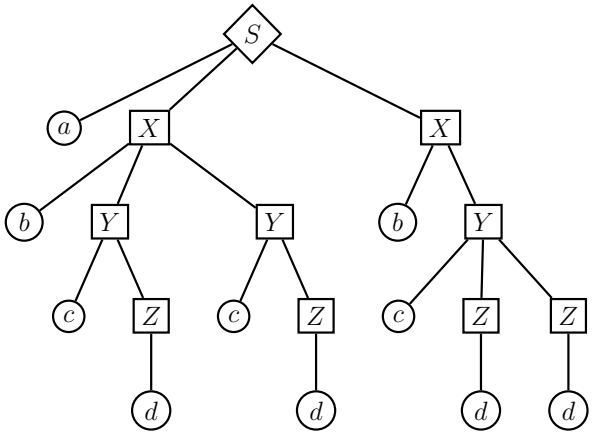

syntax tree of $a\,b\,c\,d\,c\,d\,b\,c\,d\,d$

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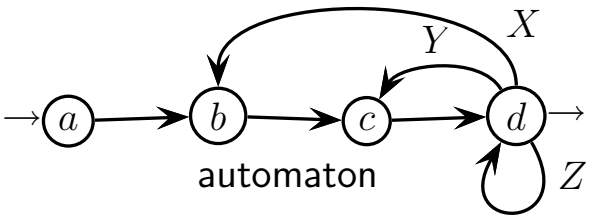

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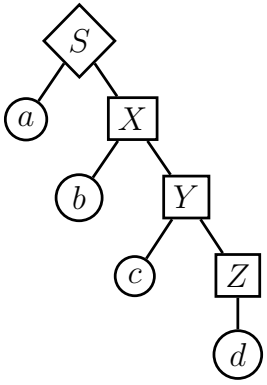

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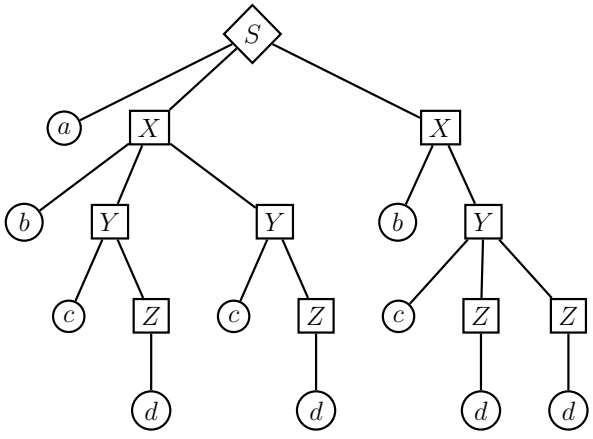


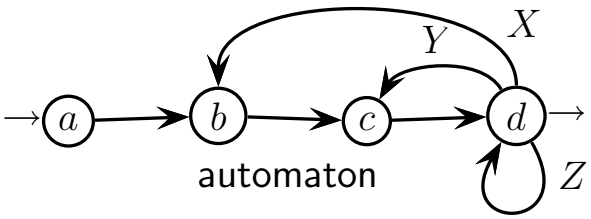
syntax tree of $a\,b\,c\,d\,c\,d\,b\,c\,d\,d$

$$\alpha_X^S = (2)$$
$$\beta_Z^Y = (1,1,2)$$

etc

integer compositions

Example



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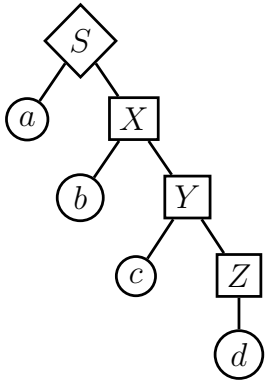

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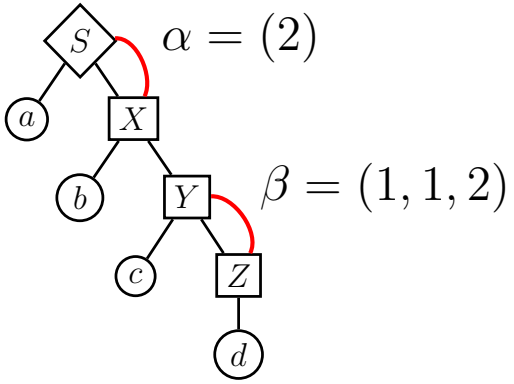
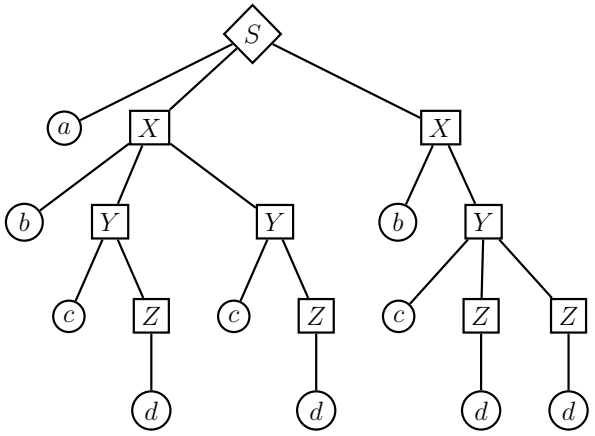


syntax tree (compact)



syntax tree of $a\,b\,c\,d\,c\,d\,b\,c\,d\,d$

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Orders on Compositions

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- suppose $Z \lhd Y \lhd X$ in a hierarchy tree

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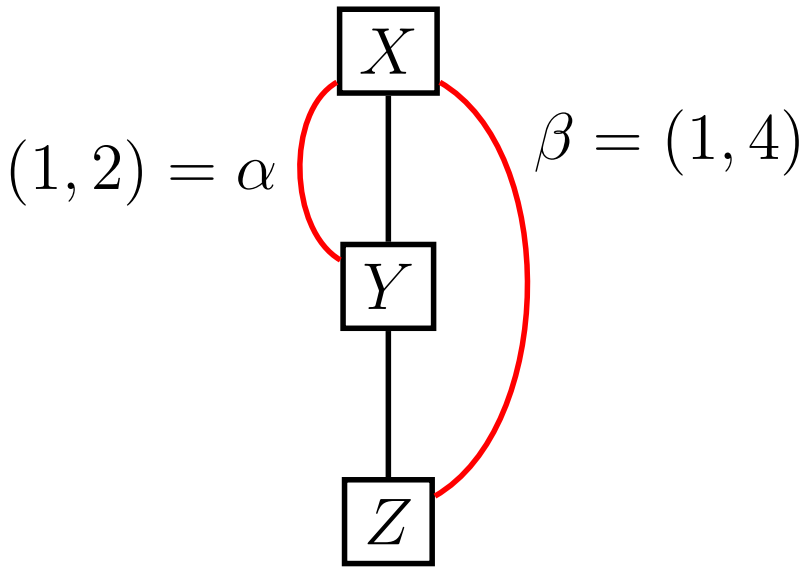

$$\alpha \leq \beta$$

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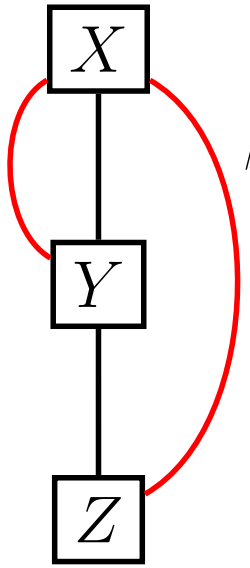
- elements of α_Z^Y are splittings of those of β_Z^X

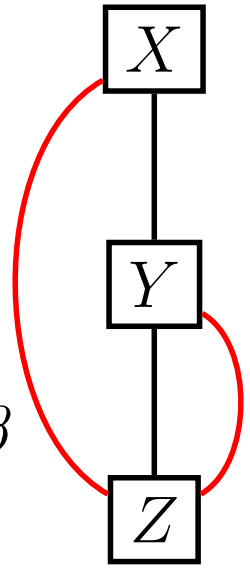
$(1,2) = \alpha$



$\beta = (1,4)$

$\alpha \leq \beta$

$(3,5) = \beta$



$\alpha = (1,2,3,2)$

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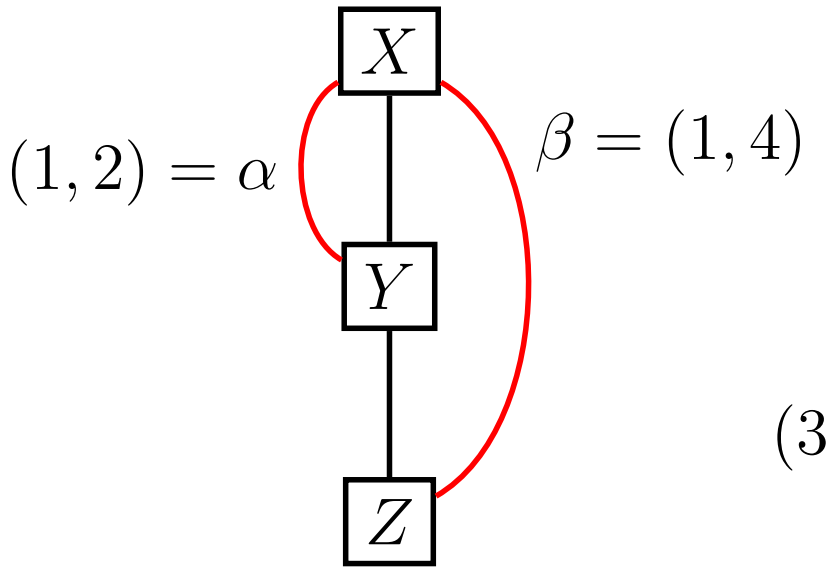

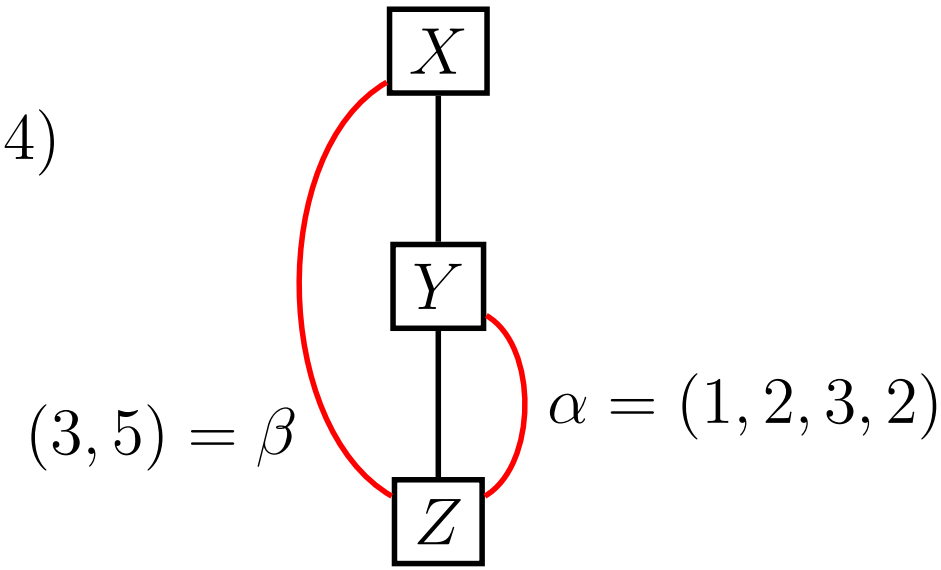

$\alpha \leq \beta$ $\qquad$ $\alpha \preceq \beta$

both relations can be verified in linear time

Operations on Compositions

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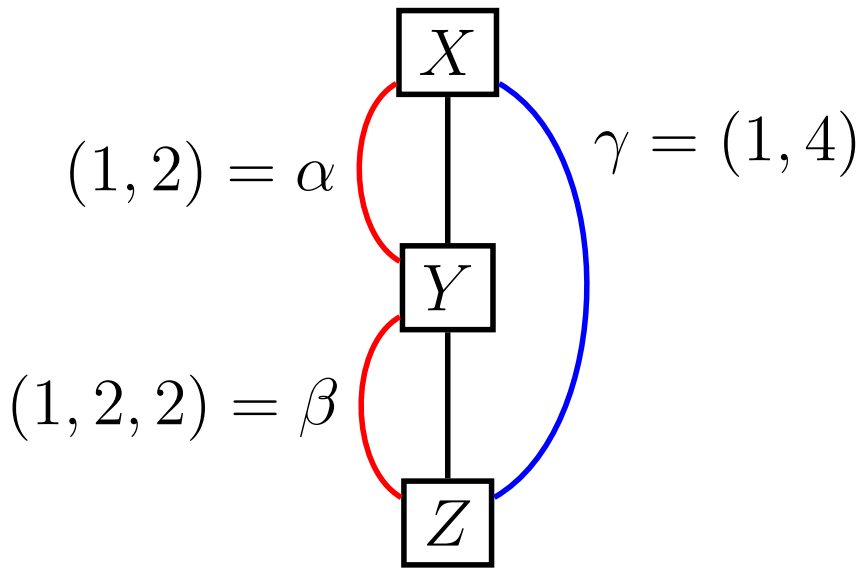

product

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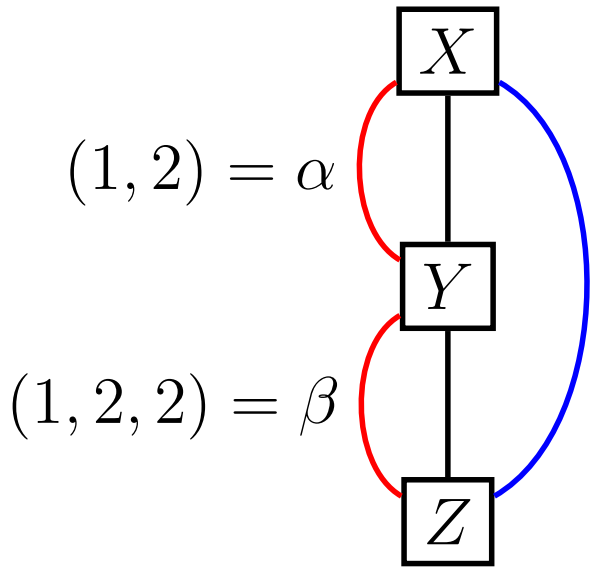
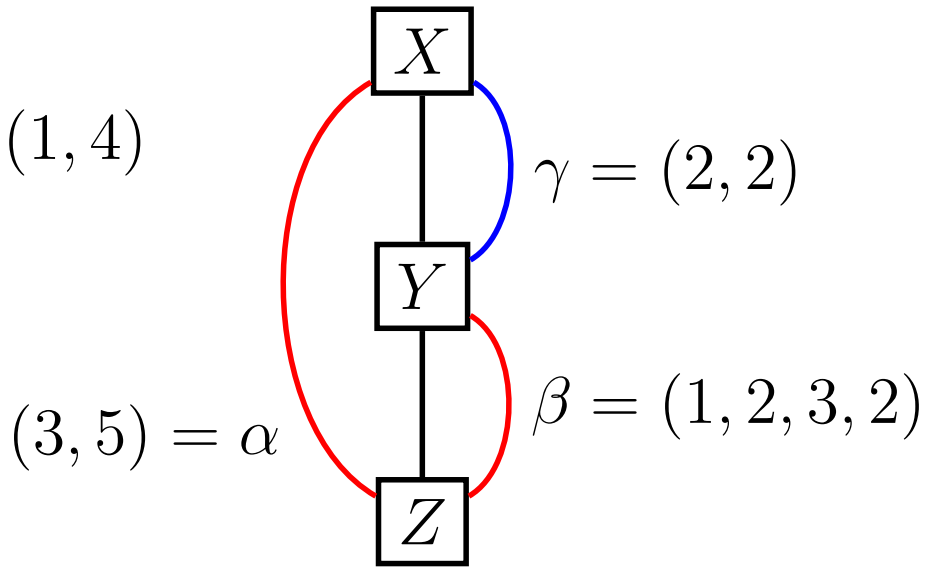
- quotient $\alpha_Z^X / \beta_Z^Y = \gamma_Y^X$ - tree relation from X downto Y



product



quotient

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- product $\alpha_Y^X \cdot \beta_Z^Y = \gamma_Z^X$ - tree relation from X downto Z
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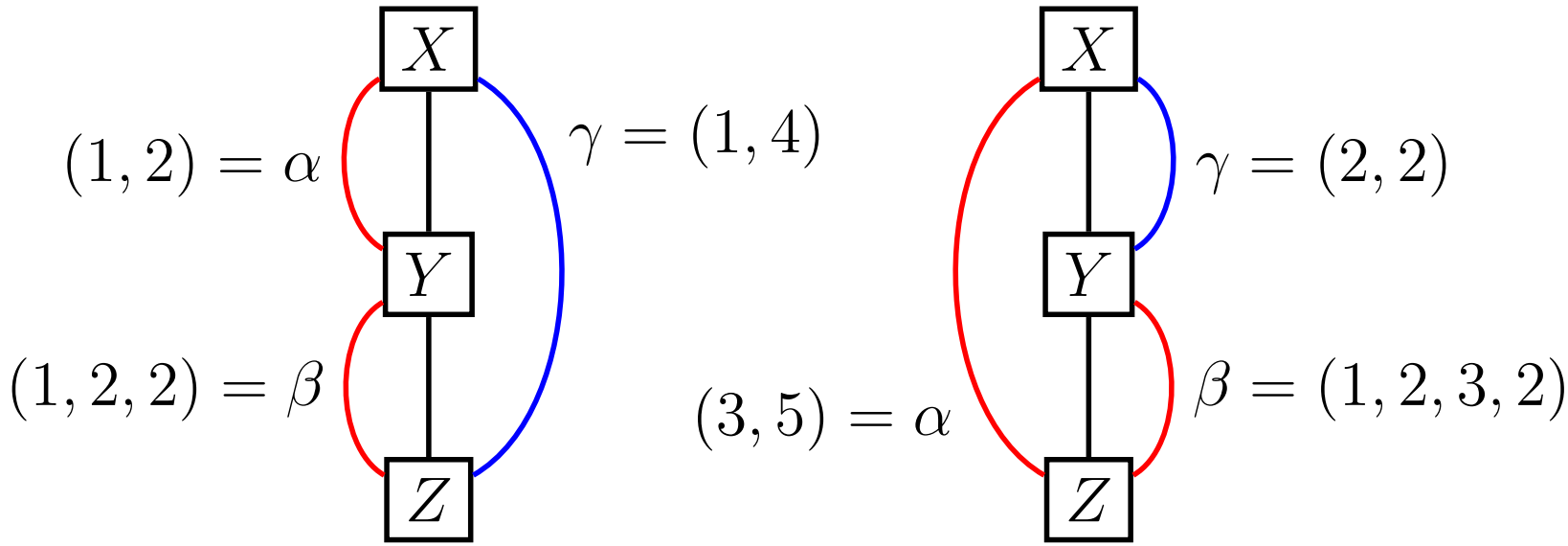

product quotient

both operations can be computed in linear time

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- a coherent set of compositions (i.e. a syntax tree) is closed with respect to both product and quotient

---

Membership Decision Procedure

Procedure Set-up

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- initial configuration of the decision procedure:

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  - loop-only automaton A

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$$y \cong_D x \ \wedge \ y \in L(A)$$

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- project string x onto the pairs of dependent letters (and onto the completely independent letters if any)
- projections of x (given) and y (to be found) are identical

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       coherently with existing comp.s - if impossible, stop and reject

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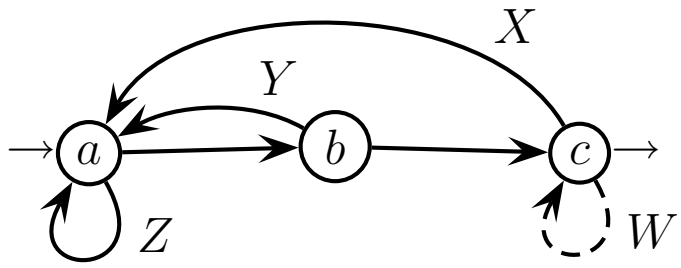
       coherently with existing comp.s - if impossible, stop and reject
     - close w.r.t. product and quotient
     - check coherence of compositions

       if coherence check fails, stop and reject

     until nothing changes any more
- now syntax tree is coherent and complete - accept

Node Count

Node Count



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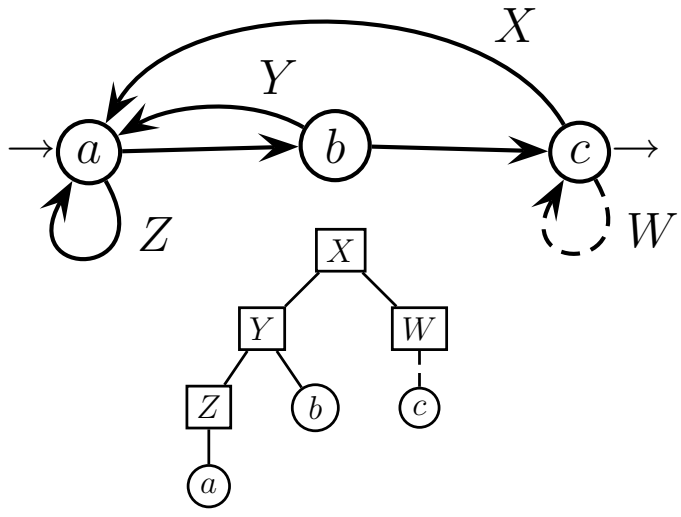

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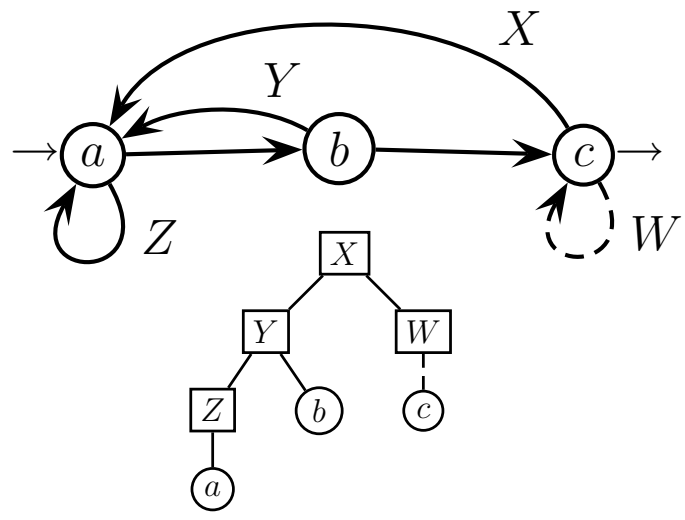

dependent letters: $a\,b$ and $a\,c$

$$y = a\,b\,a\,a\,b\,c\,a\,a\,b\,c$$

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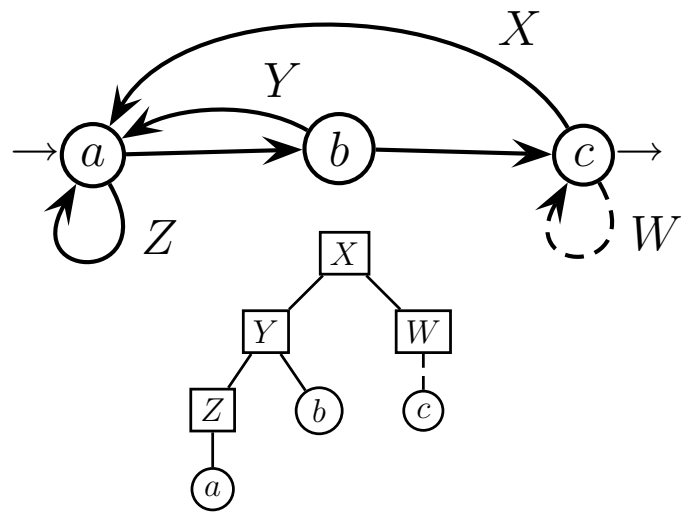

dependent letters: $a\,b$ and $a\,c$

loop	pair
X	$a\,c$
Y	$a\,b,\ b$
Z	a

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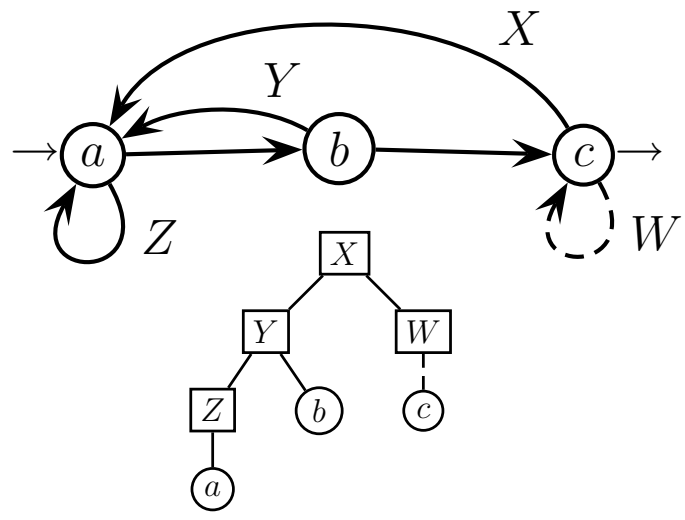

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$$\begin{aligned} \pi_{a\,c}(x) &= a\,a\,a\,c\,a\,a\,c \\ \pi_{a\,b}(x) &= a\,b\,a\,a\,b\,a\,a\,b \\ \pi_{b}(x) &= b\,b\,b \\ \pi_{a}(x) &= a\,a\,a\,a\,a \end{aligned}$$

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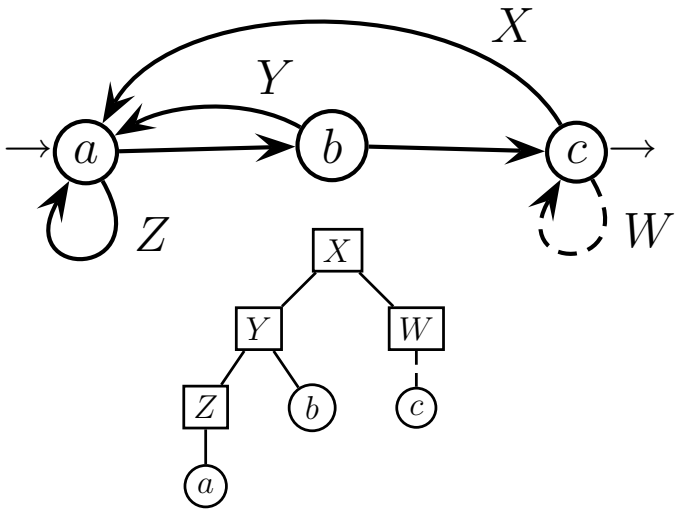

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$$\pi_{a\,c}(x) = a\,a\,a\,c\,a\,a\,c \qquad \#X = \#a\,c = 2$$
$$\pi_{a\,b}(x) = a\,b\,a\,a\,b\,a\,a\,b \qquad \#Y = \#a\,b = 3$$
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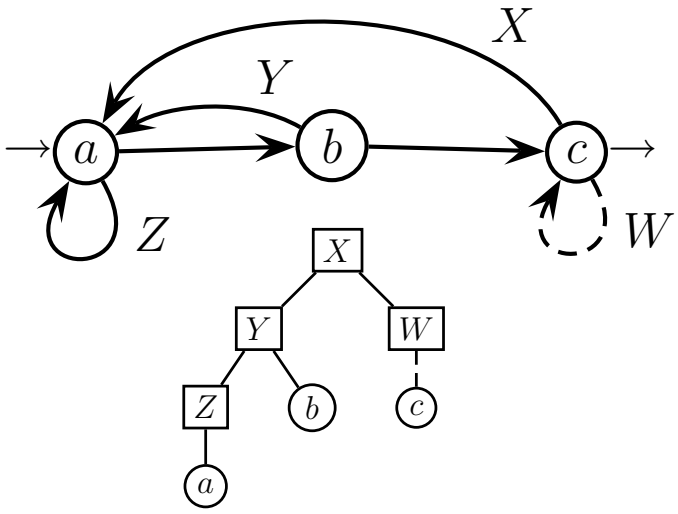

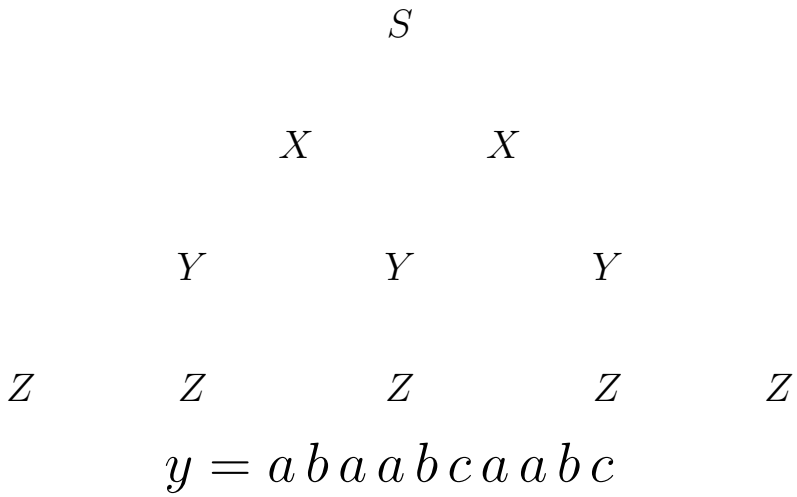

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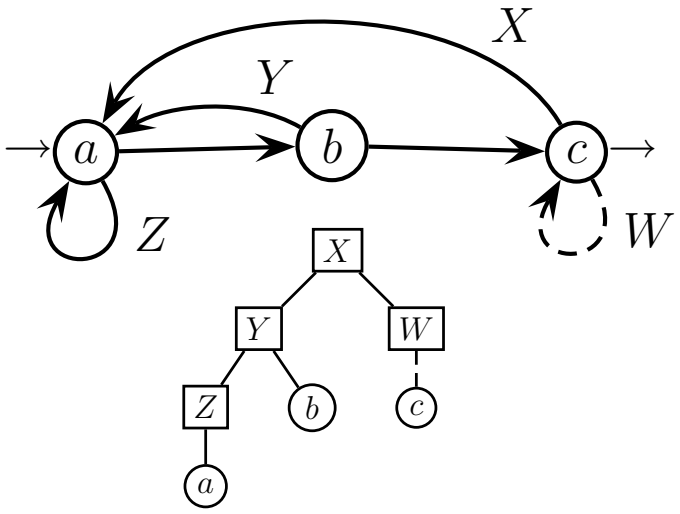

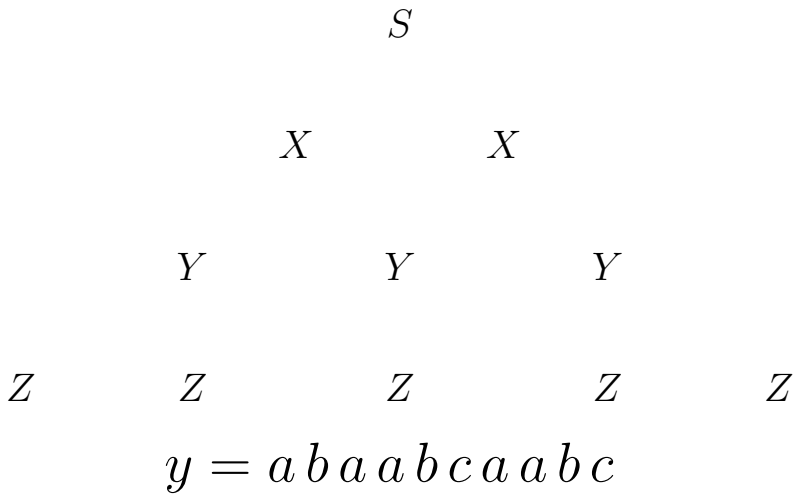

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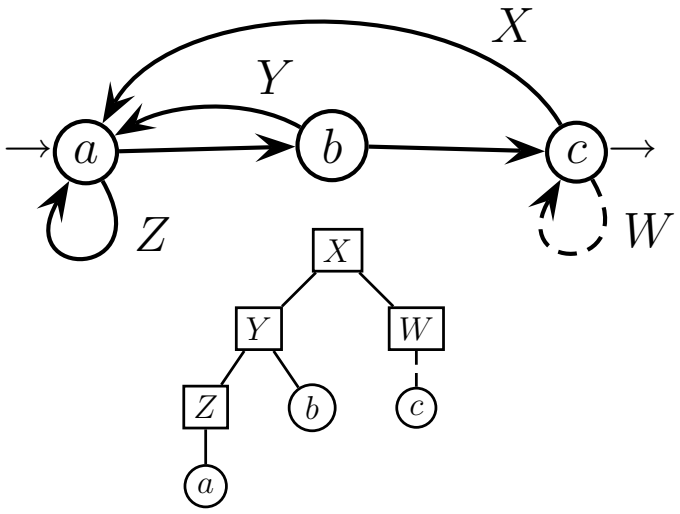
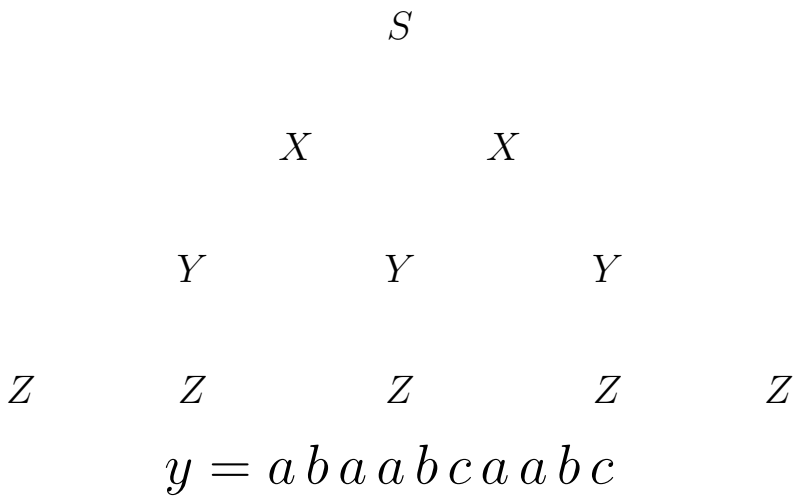
- to identify loops use dependent letter pairs

Node Count





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- to identify loops use dependent letter pairs
- for unidentified loops number of iterations is free

Composition Association

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$$\pi_{a\,c}\,(x) = a\,a\,a\,c\,a\,a\,c \qquad \chi_Z^X = (3, 2)$$

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- do so for every pair of dependent letters

Tree Closure

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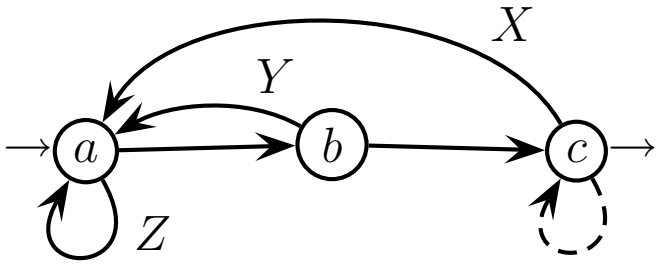

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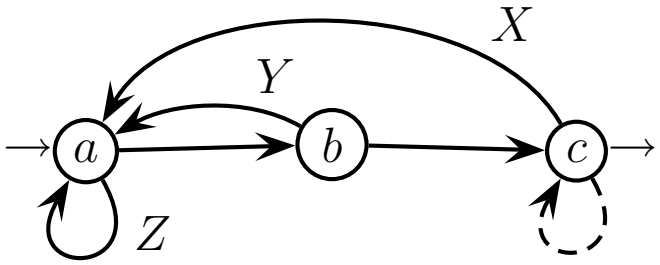

$$y = a\,b\,a\,a\,b\,c\,a\,a\,b\,c$$

$$Z \lhd Y \lhd X$$

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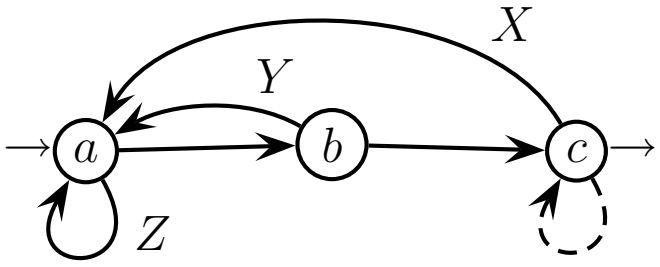
$$\chi_Z^Y = (1, 2, 2)$$

Tree Closure



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$$\chi_Z^X = (3, 2)$$

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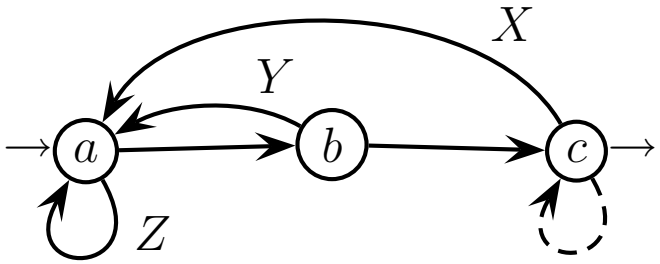
- quotient $\chi_Z^X / \chi_Z^Y = (3, 2) \,/\, (1, 2, 2) = (2, 1) = \chi_Y^X$

Tree Closure



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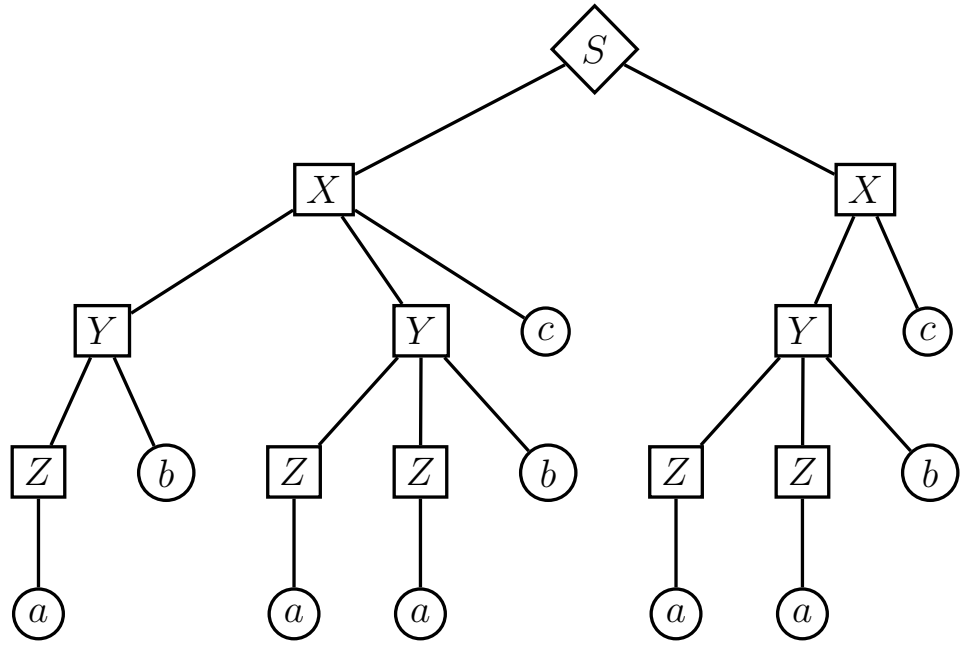

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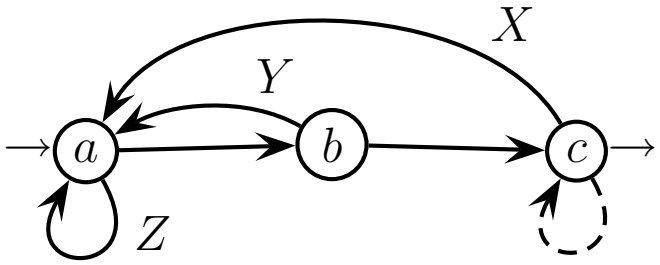

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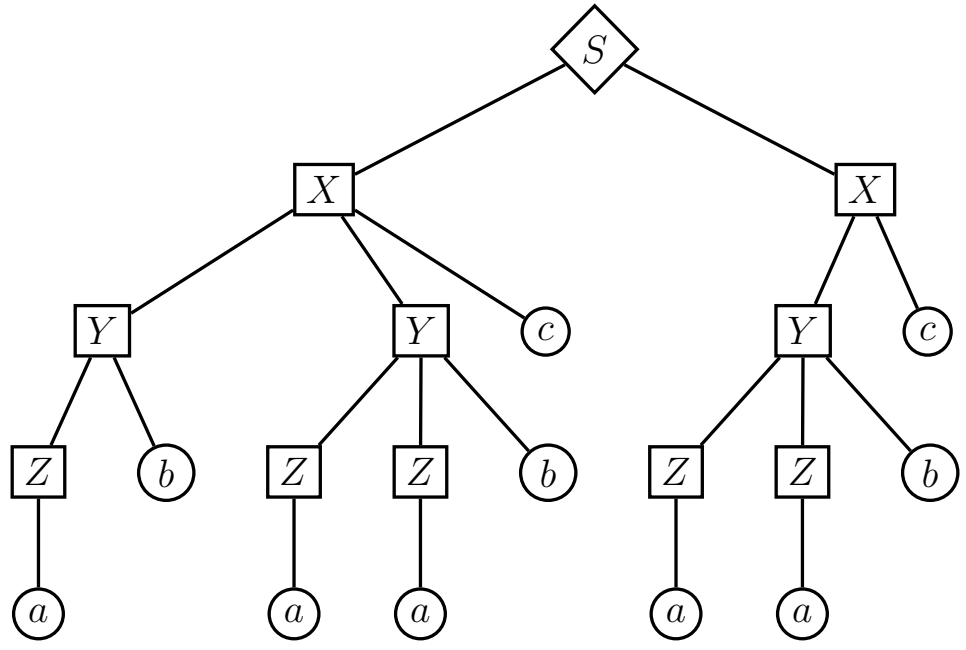

- quotient $\chi_Z^X / \chi_Z^Y = (3, 2) \,/\, (1, 2, 2) = (2, 1) = \chi_Y^X$
- tree complete - procedure end - accept string

Example with Matching

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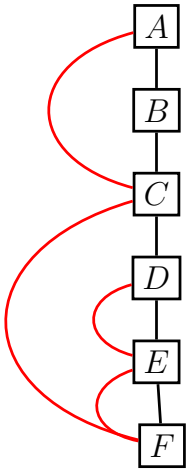
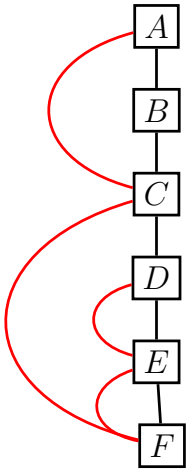
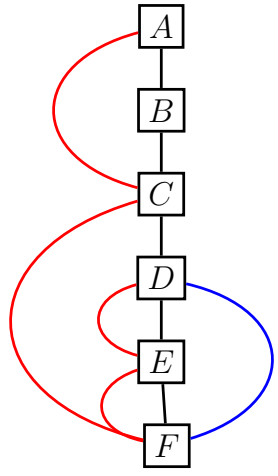


initial

Example with Matching

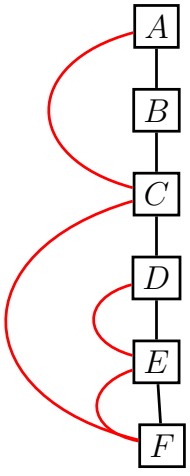


initial



product $\chi_E^D \cdot \chi_F^E = \chi_F^D$

Example with Matching



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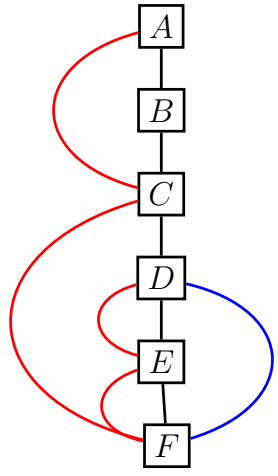

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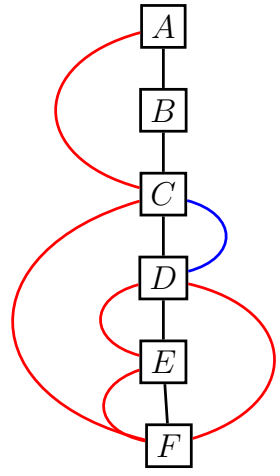

quotient $\chi_F^C / \chi_F^D = \chi_D^C$

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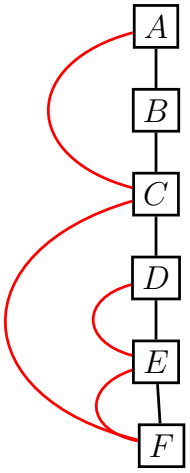

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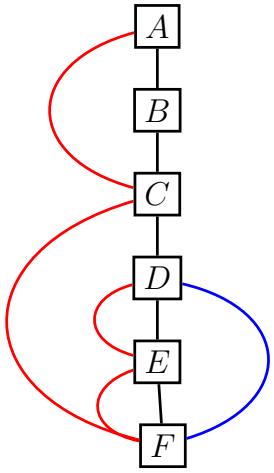

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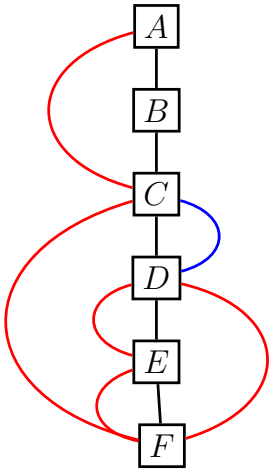

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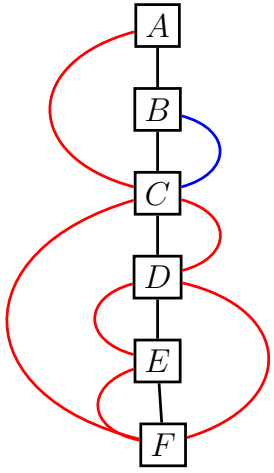

matching $\chi_C^B \preceq \chi_C^A$

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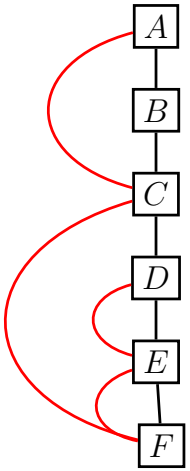
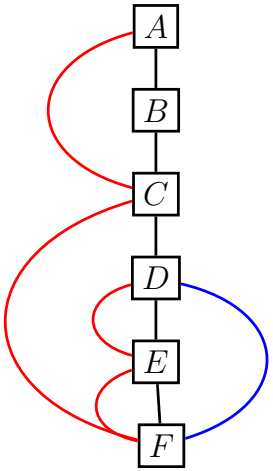


initial



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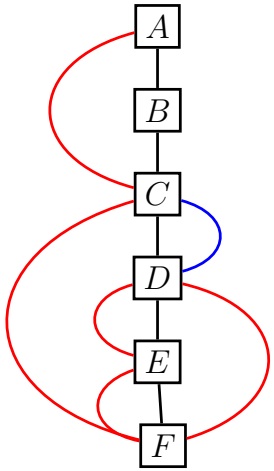

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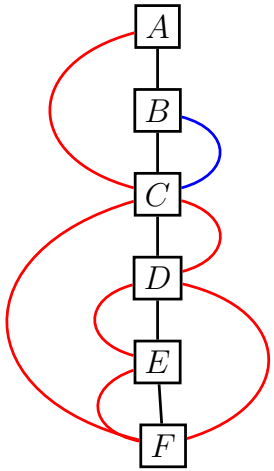

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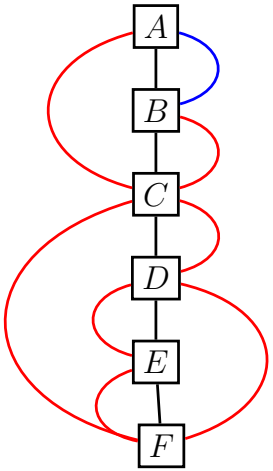

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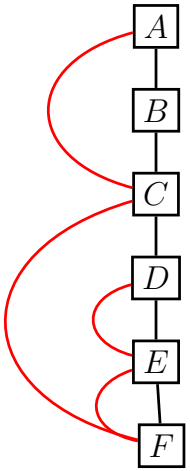

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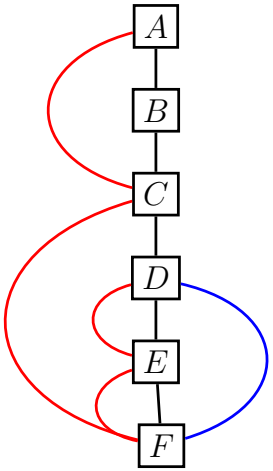

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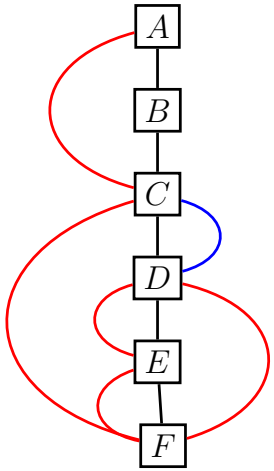

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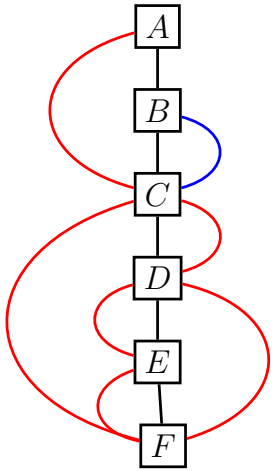

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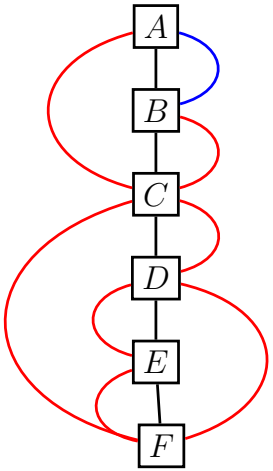

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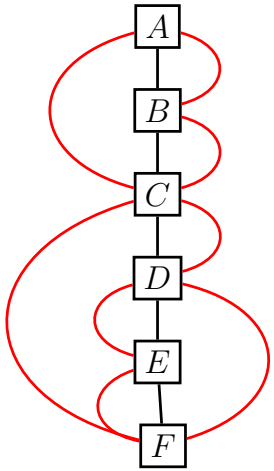

final

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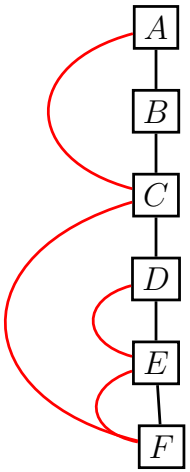

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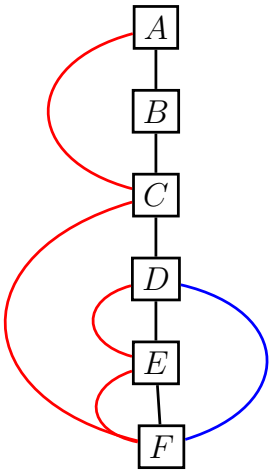

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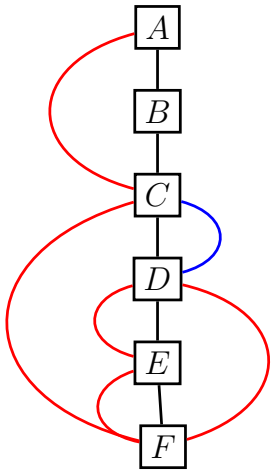

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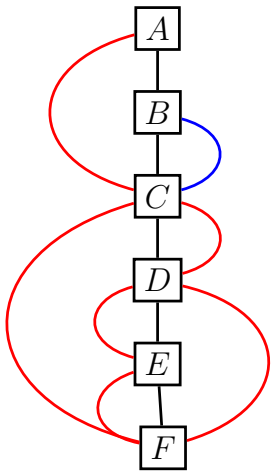

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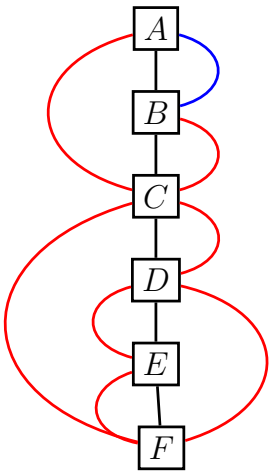

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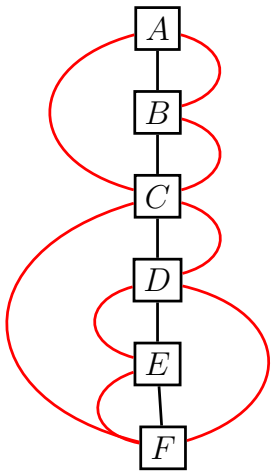

final

only the relevant products and quotients are shown

Matching Operation

What is Matching ?

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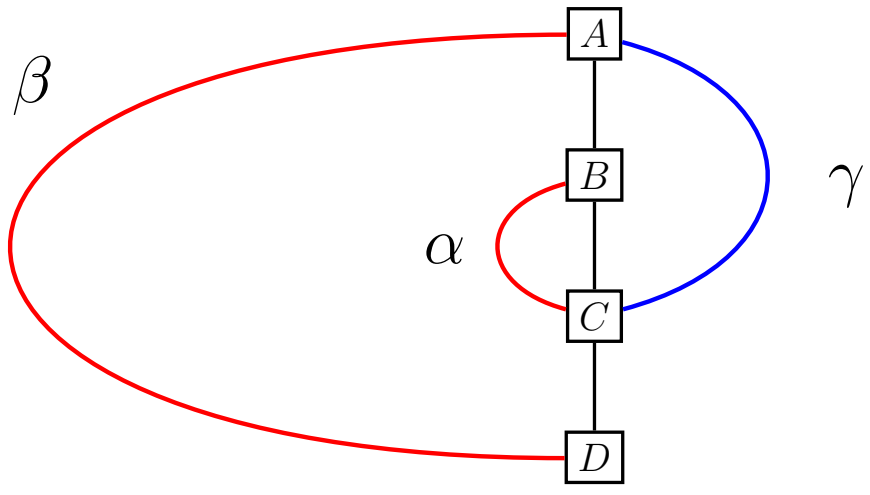

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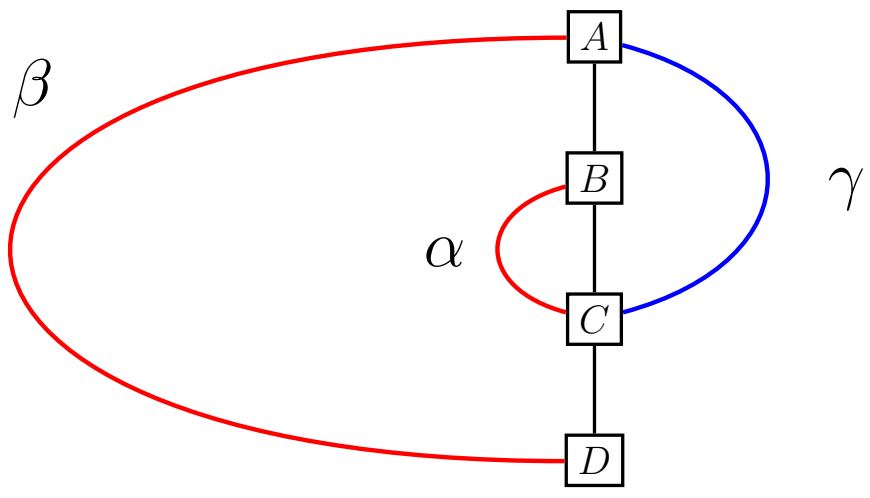

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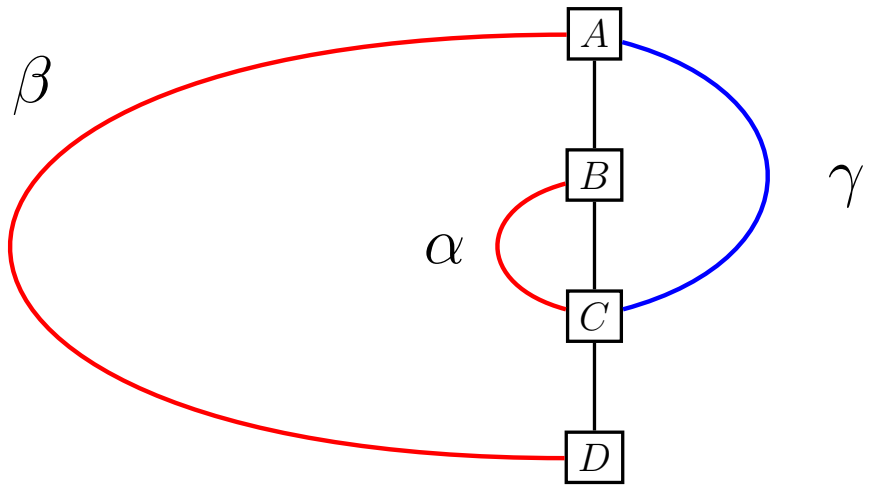

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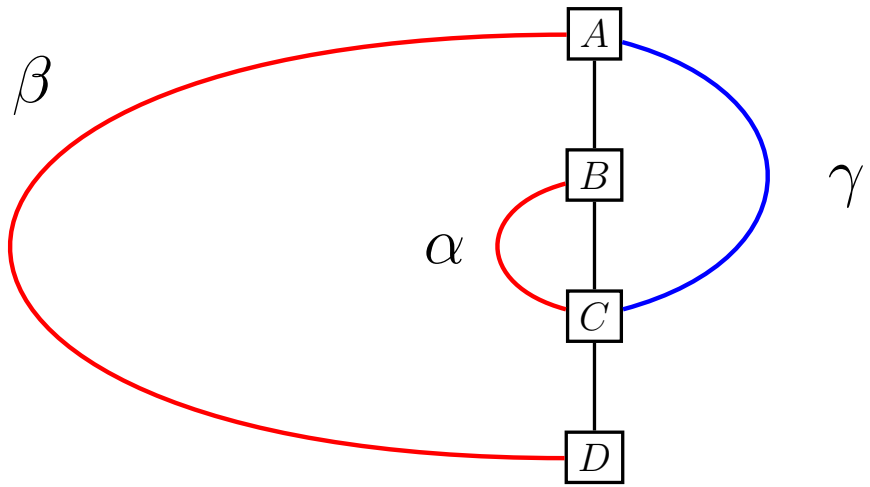

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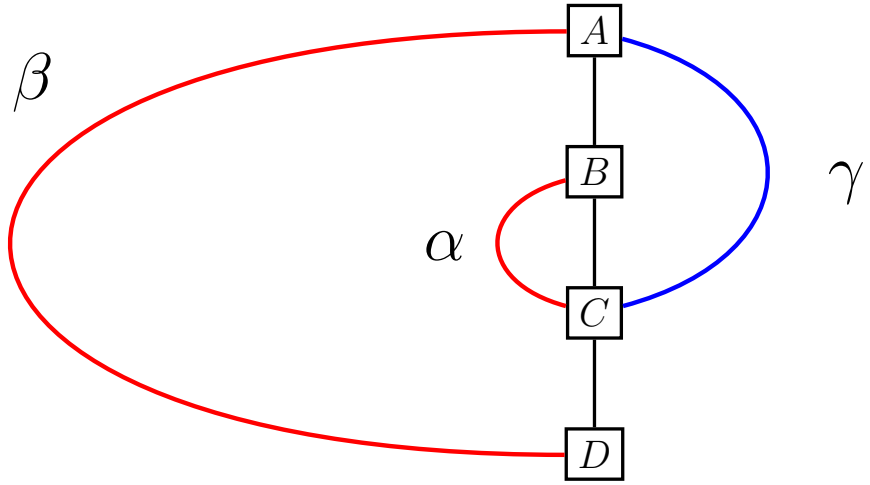

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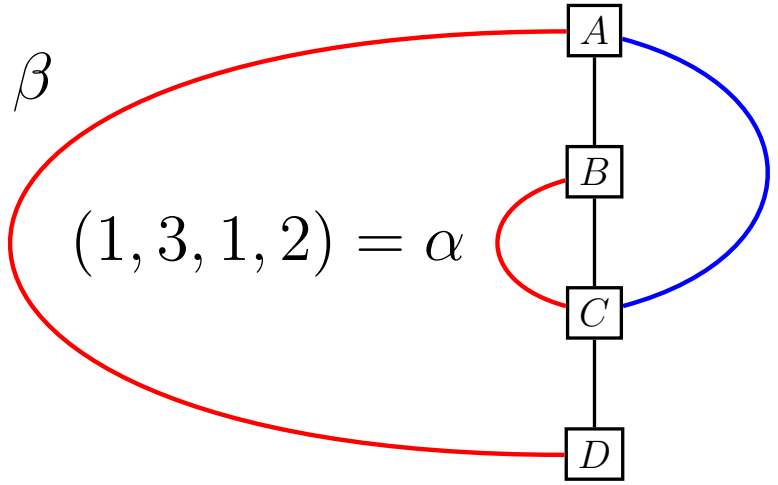

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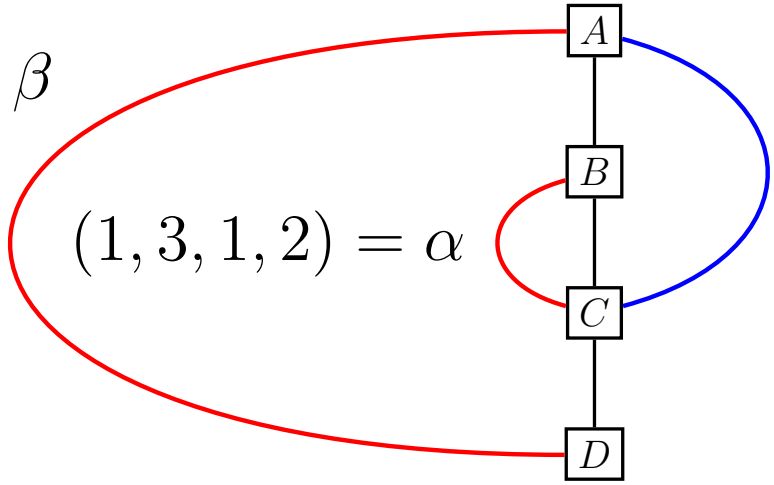

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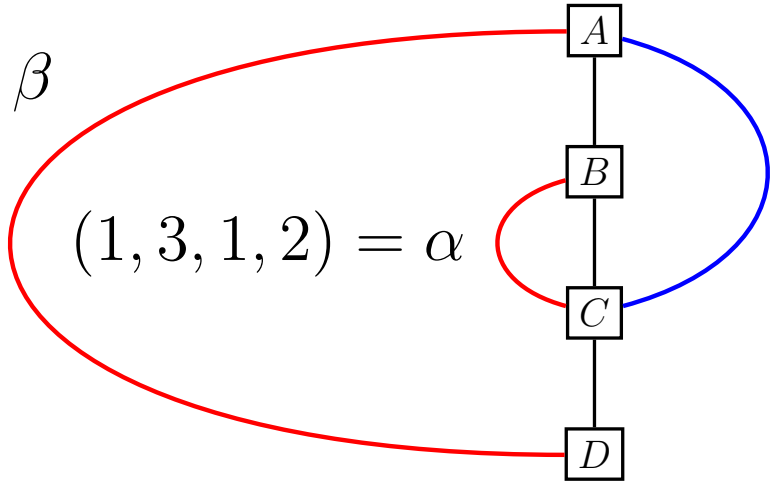

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- one can flip $\leq$ and $\preceq$ in the other three ways

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- a given integer composition can be obtained from two or even more different matching operations
- if every matching operation has either one or no result (but not two or more), then the decision procedure answers the membership problem in quadratic time
  - the procedure builds the syntax tree as well

Conclusion

Results and Open Problems

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